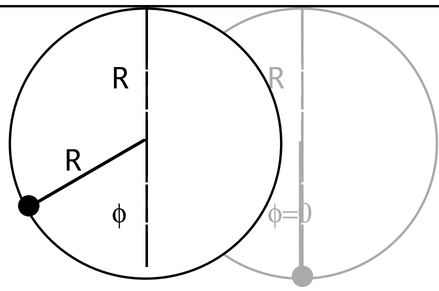
Assignment 9 - **Classical Mechanics 5103** 10/24/17 Due Tue. Oct. 31

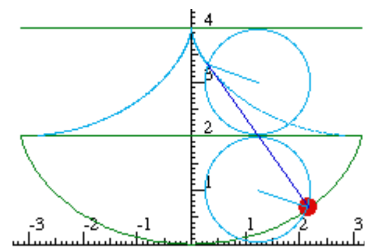
Main Reading: In new text (*Classical Mechanics with a BANG!*) Unit 2 thru 2.8 and Unit 3 thru 3.8.

Christoffel without Christawful

Ex.**1.** The spherical coordinate metric (Ex.1 Set 8) could be used to derive the Christoffel fictitious-force coefficients $\Gamma_{ij;k}$ (1st kind) and $\Gamma_{ij;}{}^{k}$ (2nd kind) by equation (3.6.10). An easier way is to use the "4-wheel drive" form of Lagrange derivative equations (2.4.1) or covariant force Lagrange equations (3.5.10) also described in Lect. 17. Use this to derive both kinds of Christoffel Γ coefficients and discuss what fictitious-forces or accelerations they give.

Huygen's problem. For 40 years Christian Huygens worked to improve harmonicity of pendulums and only solved the problem you are about to do, just before he died. Let's hope it doesn't take <u>*you*</u> *as long!* *(cackle! cackle!)*





A really scary Halloween roller coaster *(cackle! cackle!)*

Ex.**2.** A mass m slides frictionlessly along a cycloid of radius $R=\frac{3}{\pi}\,cm.$ in gravity $g\sim 10m\cdot s^{-2}$. Or else, a very light hoop is held up against a magnetized ceiling so it rolls with no slip carrying mass m on its rim. (See sketch).

(a_1) First construct *24pt* normal cycloids using ruler-compass on graph paper. (End of Lect.17 has a 6*cm.* graph whose *x-axis* has 24 intervals that "roll" circles of radii R=3/π*cm.* by angles $n\pi/12$ above and below and π out of phase. Check geometry with algebra.))

(a_2) Write parametric equations $x(\phi)$, $y(\phi)$ for a circumference point mass on a wheel rolling on the ceiling where ϕ is the wheel-center-relative angle that mass has rotated from its lowest point at lab origin $(x=0,y=0)$.

(b) Derive Lagrangian $L(d\phi/dt,\phi)$ and find canonical momentum p_ϕ and equation of motion.

(c) Derive total energy and Hamiltonian function H. Are any of these *(L, p_ϕ,E, or H) ever* constants of motion?

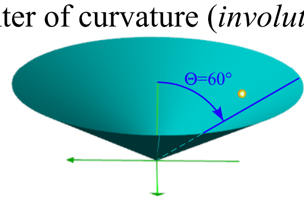
(d) Derive an expression for the arc length $s(\phi)$ that m travels to ϕ from the lowest point $\phi=0$. How long is a string wrapped around a cycloid from $\phi=0$ to $\phi=\pi$.

(e) Prove period of oscillation of mass m is independent of initial velocity $\dot{\phi}(0)$ at $\phi=0$ for $\dot{\phi}(0)$ less than_______?

What is that max $v_{\max}=\dot{x}_{\max}$ =____? and max $\omega_{\max}=\dot{\phi}_{\max}$ =____? (Discuss and give both algebraic and numerical results.)

(f) Derive $\phi(t)$ for free oscillation. Can the hoop roll across the ceiling with constant speed $v_{const}=\dot{x}_{const}$ for some initial conditions ? Discuss. Does such a uniform velocity state exist if instead the hoop rolls on the floor?

(g) Show how a cycloid path is generated by a string unwrapping (*evolute*) from another cycloid and that a normal cycloid is the locus of the center of curvature (*involute*) of a π-out-of-phase cycloid like itself. (Use (a) to discuss.)



Ex.**3.** *Funny funnel orbits*

(a) Analyze orbits for mass sliding on funnel cone of polar angle Θ in gravity $g_z\sim 10m\cdot s^{-2}$. (Recall "I-Ball" in Lecture 17.) What closed and periodic orbit (if any) listed in Fig. 3.8.1 is closest to being achieved for Θ=60°? ω_r/ω_ϕ=______?

(b) Find Θ-cones that do give closed orbits with frequency ratios given below and (if orbit is possible) sketch its path.

$\omega_r/\omega_\phi=1/1$: $\Theta_{1/1}$=___; $\omega_r/\omega_\phi=2/1$: $\Theta_{2/1}$=___; $\omega_r/\omega_\phi=3/1$: $\Theta_{3/1}$=___;

$\omega_r/\omega_\phi=1/2$: $\Theta_{1/2}$=___; $\omega_r/\omega_\phi=3/2$: $\Theta_{3/2}$=___; $\omega_r/\omega_\phi=1/3$: $\Theta_{1/3}$=___; $\omega_r/\omega_\phi=2/3$: $\Theta_{2/3}$=___.