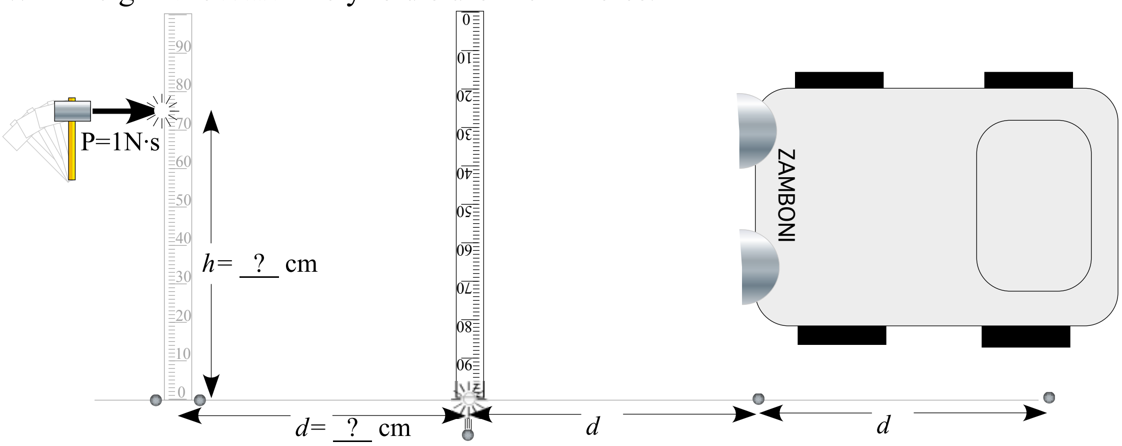
Assignment 9 - **Classical Mechanics 5103** 10/27/16 Due Thur Nov. 3

Main Reading: In new text (*Classical Mechanics with a BANG!*) Unit 2 thru 2.9 and Unit 3 thru 3.8.

An icy cycloid problem

2.A.1 (a) A meter stick lies on a smooth icy hockey rink surface with two marbles sitting at its end on either side of the 0.0cm mark. (See figure) A hammer give impulse $\mathbf{P}=(1\text{N}\cdot\text{s})\mathbf{e_x}$ to the stick at the h-cm. mark.
What height h is *least* likely to disturb the marbles.



(b) Now assume h-value from (a) and friction-free "icy" surface. At what distances d, $2d$, $3d$, … along x-axis should the 3^{rd}, 4^{th}, 5^{th},…marbles be placed so they are most likely to be knocked below the axis. Draw 6 equal time Δt interval snapshots of the stick as it flips by 180° and then to 360°. What is Δt for a 1kg stick?

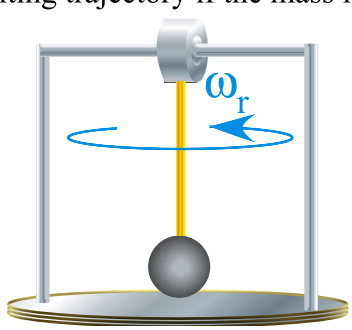
Electromagnetic cycloids

2.8.1 Suppose a unit mass $m=1$ kg and charge $Q=1$ $Coul.$ (Dangerous!) is dropped from ($x=0=y$) along a vertical frictionless (x,y)-surface in Earth gravity (Say $g_y=-10m/s^2$) with in a strong z-axial magnetic $\mathbf{B}_z$-field.

(a) How many Tesla of magnetic field $\mathbf{B}_z$ and in what direction (±) would cause the mass to move toward $+x$ on a normal cycloid made by circle of one meter diameter? Where would it again touch the horizontal x-axis?

(b) What initial speed and direction of throw would cause the mass to fly straight along the $+x$-axis?

(c) Describe and plot the resulting trajectory if the mass is thrown down with a speed of $2m/s$.



Pendulum on turntable

3.8.5 Suppose a pendulum supported by a circular ball bearing may swing without friction in the vertical plane of the bearing. The bearing plane is secured to a turntable that rotates at a constant angular frequency ω_r. The pendulum consists of a mass m at the end of a rod of length $\ell=1m$ and negligible mass with natural frequency of small θ-angle motion at zero-ω_r in gravity acceleration (Say $g=10m/s^2$) given by $\omega_0(\omega_r=0)$=___.

(a) Derive the Lagrangian and Hamiltonian using spherical coordinates in the rotating frame.

(b) Derive the θ-equilibrium points and small-oscillation frequency as a function of the frequency ω_r and ω_0. Overlay plots of effective θ-potential for several key values of ω_r. What ω_r value makes $\theta=0$ angle unstable?