Assignment 8 Oct 23 , 2019 Due Wednesday Oct 30: Based on Unit 2 Chapter 1-3 and Unit 3 Chapter 1-3.

Well-known Coordinates (OCC) *NOTE: Save copy of solution to this Ex.***1**(b) *for next Assignment 9.*

1. Find Jacobian, Kajobian, $\mathbf{E}_m$, $\mathbf{E}^m$, metric tensors g_{mn} and g^{mn} for OCC (a) and (b). (*You may do (b) then reduce to (a).*)

(a) Cylindrical coordinates $\{q^1=\rho, q^2=\phi, q^3=z\}$: $x=x^1=\rho\cos\phi$, $y=x^2=\rho\sin\phi$, $z=x^3$.

(b) Spherical coordinates: $\{q^1=r, q^2=\theta, q^3=\phi\}$: $x=x^1=r\sin\theta\cos\phi$, $y=x^2=r\sin\theta\sin\phi$, $z=x^3=r\cos\theta$.

"Plopped" Parabolic Coordinates (GCC) *(In attached figure)*

2. Consider the GCC(Cartesian) definition: $q^1 = (x)^2 + y \quad q^2 = (y)^2 - x$

(a) Does an analytic Cartesian coordinate definition $x^j = x^j(q^m)$ exist? If so, show.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the appropriate graph on attached pages sketch the unitary vectors at the point $(x=1, y=1)$ (Arrow) and at the point $(x=1, y=0)$. Where, if anywhere, is the grid an OCC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there Jacobian or Kajobian singularities of this GCC. Show on graph.

"Sliding" Parabolic Coordinates (GCC) *(In attached figure)*

3. Consider the Cartesian(GCC) definition: $x = 0.4\,(q^1)^2 - q^2\,, \quad y = q^1 - 0.4\,(q^2)^2$

(a) Does an analytic GCC coordinate definition $q^m = q^m(x^j)$ exist? If so, show.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the appropriate graph on attached pages sketch the unitary vectors near point $(x=1, y=1)$ (Arrow) and near point $(x=1, y=0)$. Where, if anywhere, is the grid an OCC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there Jacobian or Kajobian singularities of this GCC. Show on graph.

4. *Covariant vs Contravariant Geometry* *(In attached figure)*

GCC components of a vector **V** in attached figure are realized by line segments OA, BV, etc. Give each segment length by single terms of the form V_m or V^m times $(\sqrt{g_{mm}})^{+1}$, $(\sqrt{g_{mm}})^{-1}$, $(\sqrt{g^{mm}})^{+1}$, or $(\sqrt{g^{mm}})^{-1}$ with the correct $m=1$ or 2. Also label each unitary vector as $\mathbf{E}_1$, $\mathbf{E}^1$, $\mathbf{E}_2$, or $\mathbf{E}^2$, whichever it is.

You should be able to do this quickly without looking at the text figures.

Extra Credit 3D problem: *"Unprofessional" Paraboloidal Coordinates (GCC)* *(In attached figure)*

5. The surface $z = f(x,y) = \frac{1}{2}x^2 + y^2$ (See *xyz*-plot) introduces 3D partial derivative chain rules. It is the ($q^3=0$)-surface in a 3D GCC coordinate grid $q^1=x,\ q^2=y,\ q^3=\frac{1}{2}x^2+y^2-z$. It contains a projection of an orthogonal (x,y) Cartesian coordinate grid on the surface that is obviously *not* orthogonal most places.

a. Derive the *3-by-3* Jacobian $J(x,y,z)$ and Kajobian $K(x,y,z)$ for ($q^3=0$).

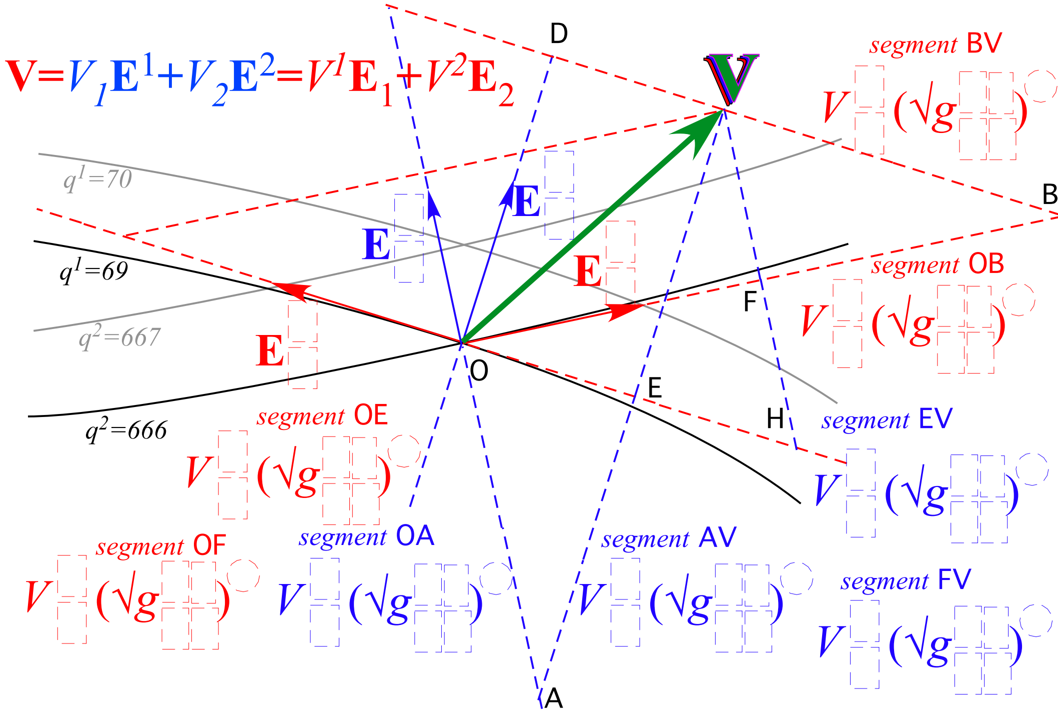
b. Extract covariant $\{\mathbf{E}_1,\mathbf{E}_2,\mathbf{E}_3\}$ and contravariant $\{\mathbf{E}^1,\mathbf{E}^2,\mathbf{E}^3\}$ vectors represented in Cartesian (x,y,z) basis.

c. Derive the *3-by-3* covariant metric $g_{\upsilon\nu}(x,y)$ and contravariant metric $g^{\upsilon\nu}(x,y)$ for ($q^3=0$) and tell which if any points on the surface have grids that are locally *orthogonal* and which if any are locally *orthonormal*.

*(Larger graph provided separately for Ex.***5***d and Ex.***5***e.*

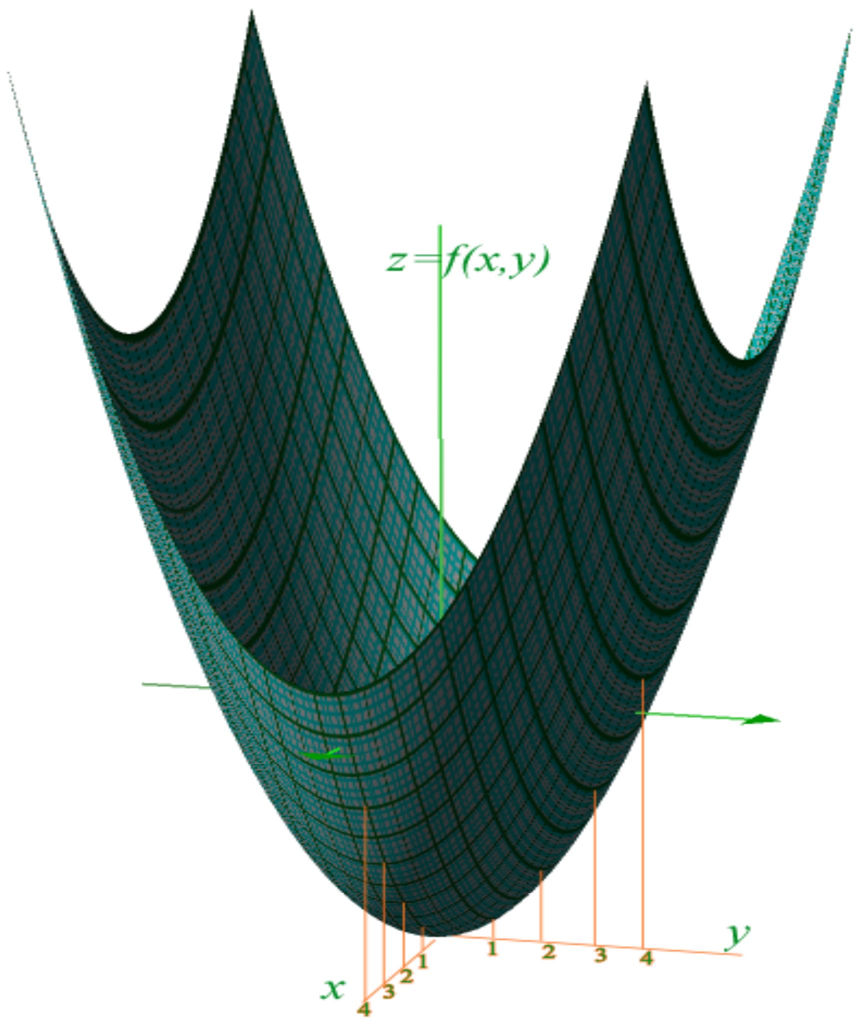
d. Calculate and sketch covariant $\{\mathbf{E}_1,\mathbf{E}_2,\mathbf{E}_3\}$ on ($q^3=0$) surface where $(x=4,y=-2)$ and where $(x=3,y=+2)$.

e. Calculate and sketch contravariant $\{\mathbf{E}^1,\mathbf{E}^2,\mathbf{E}^3\}$ on ($q^3=0$) surface where $(x=4,y=+2)$ and where $(x=0,y=+4)$.

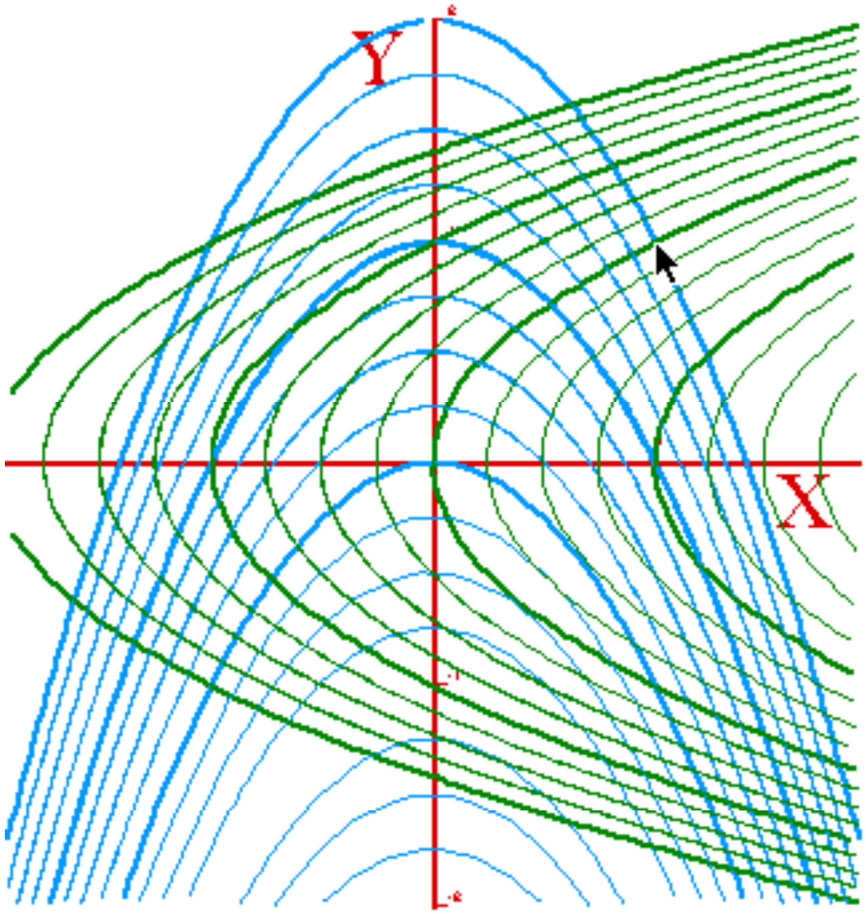


Above is 2D plot for *Ex.* **4**

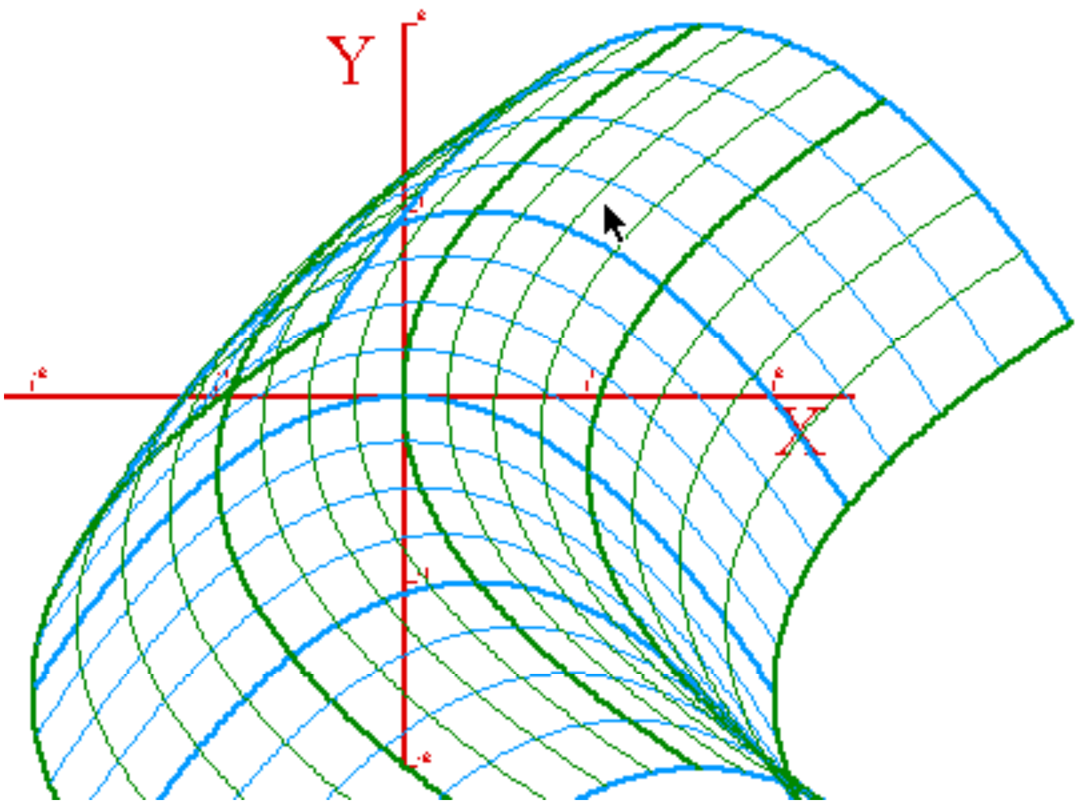
*Below is 3D (xyz) plot for Extra credit Ex.***5**.



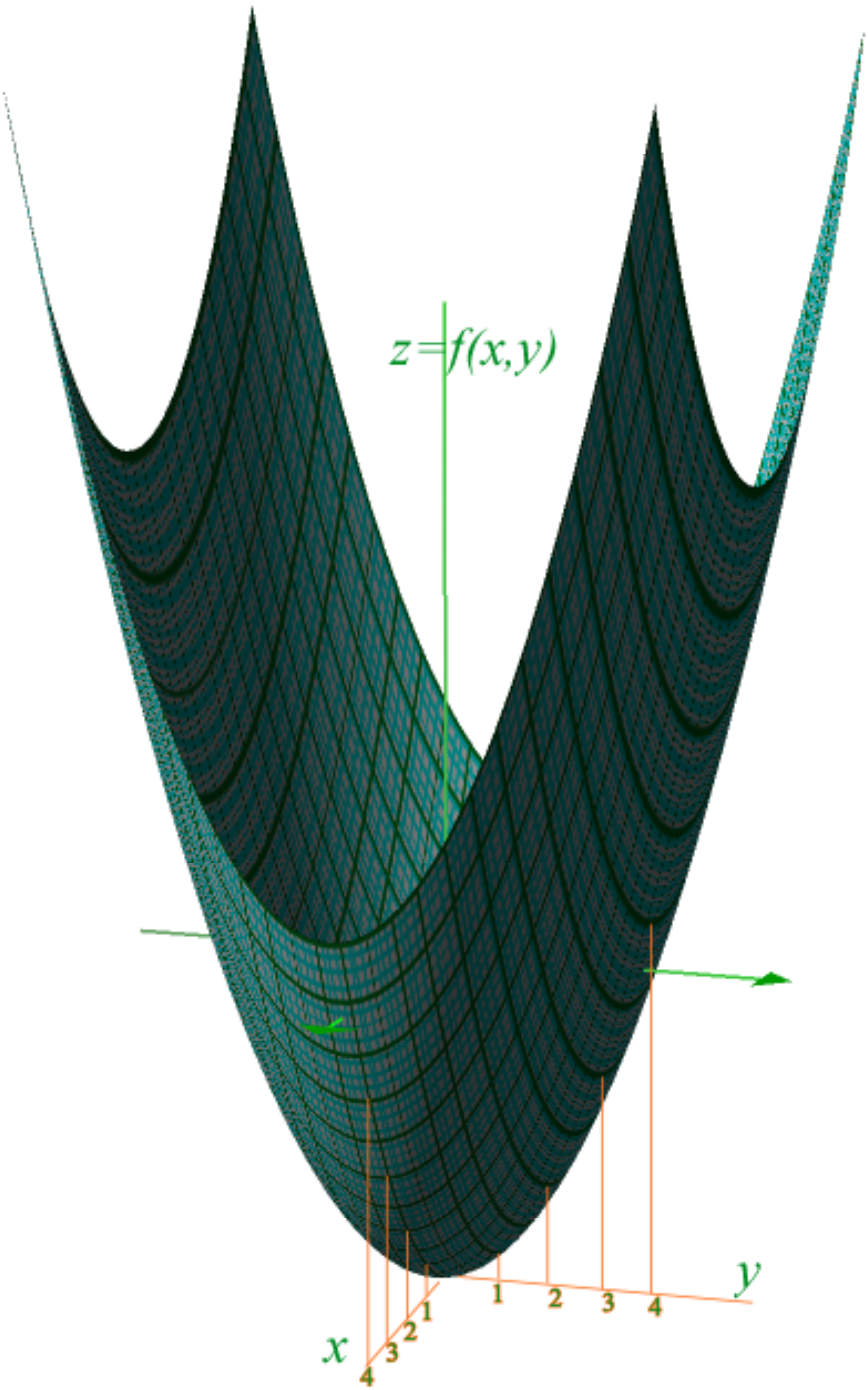
*"Plopped" and"Sliding" Parabolic Coordinates are 2D (xy) plots for Ex.***2** *and Ex.***3** *(despite 3D* *__appearance__* *(only) of latter.)*



*"Plopped" Parabolic Coordinates for Ex.***2**



*"Sliding" Parabolic Coordinates for Ex.***3**



Assignment 8 (contd.) - Extra credit Ex. **5**
"Unprofessional" Paraboloidal Coordinates

Assignment 8 Solutions

Ex.1 Compute Jacobian, Kajobian, $\mathbf{E}_m$, $\mathbf{E}^m$, metric tensors g_{mn} and g^{mn} for the following OCC.

(c) Cylindrical coordinates $\{q^1=\rho,\ q^2=\phi\}$: $x=x^1=\rho\cos\phi$, $y=x^2=\rho\sin\phi$.

Spherical coordinates: $\{q^1=r,\ q^2=\theta,\ q^3=\phi\}$: $x=x^1=r\sin\theta\cos\phi$, $y=x^2=r\sin\theta\sin\phi$, $z=x^3=r\cos\theta$.

3.6.1 Jacobian, Kajobian, $\mathbf{E}_m$, $\mathbf{E}^m$, metric g_{mn} and g^{mn} for spherical coordinates and cylindrical coordinates

Spherical coordinates: $\{q^1=r,\ q^2=\theta,\ q^3=\phi\}$: $x=x^1=r\sin\theta\cos\phi$, $y=x^2=r\sin\theta\sin\phi$, $z=x^3=r\cos\theta$, reduce to cylindrical coordinates $\{q^1=\rho,\ q^2=\phi\}$: $x=x^1=\rho\cos\phi$, $y=x^2=\rho\sin\phi$ for $\rho=r$ and $\theta=\pi/2$: (So spherical coordinates are detailed first below.)

Jacobian matrices and determinants:

$$J=\overset{\quad\mathbf{E}_r\qquad\mathbf{E}_\theta\qquad\mathbf{E}_\phi}{\begin{pmatrix}\frac{\partial x}{\partial r}&\frac{\partial x}{\partial\theta}&\frac{\partial x}{\partial\phi}\\ \frac{\partial y}{\partial r}&\frac{\partial y}{\partial\theta}&\frac{\partial y}{\partial\phi}\\ \frac{\partial z}{\partial r}&\frac{\partial z}{\partial\theta}&\frac{\partial z}{\partial\phi}\end{pmatrix}}=\begin{pmatrix}\sin\theta\cos\phi&r\cos\theta\cos\phi&-r\sin\theta\sin\phi\\ \sin\theta\sin\phi&r\cos\theta\sin\phi&r\sin\theta\cos\phi\\ \cos\theta&-r\sin\theta&0\end{pmatrix}\xrightarrow[r=\rho]{\theta=\pi/2}\begin{pmatrix}\cos\phi&0&-\rho\sin\phi\\ \sin\phi&0&\rho\cos\phi\\ 0&-\rho&0\end{pmatrix}$$

$$\det J=\det J^T=\frac{\partial\{xyz\}}{\partial\{r\theta\phi\}}=r^2\sin\theta\xrightarrow[r=\rho]{\theta=\pi/2}\rho^2$$

"Kajobian" matrix inverses of J.

$$K=J^{-1}=\begin{pmatrix}\frac{\partial r}{\partial x}&\frac{\partial r}{\partial y}&\frac{\partial r}{\partial z}\\ \frac{\partial\theta}{\partial x}&\frac{\partial\theta}{\partial y}&\frac{\partial\theta}{\partial z}\\ \frac{\partial\phi}{\partial x}&\frac{\partial\phi}{\partial y}&\frac{\partial\phi}{\partial z}\end{pmatrix}=\frac{\begin{pmatrix}\begin{vmatrix}r\cos\theta\sin\phi&r\sin\theta\cos\phi\\-r\sin\theta&0\end{vmatrix}&-\begin{vmatrix}r\cos\theta\cos\phi&-r\sin\theta\sin\phi\\-r\sin\theta&0\end{vmatrix}&\begin{vmatrix}r\cos\theta\cos\phi&-r\sin\theta\sin\phi\\r\cos\theta\sin\phi&r\sin\theta\cos\phi\end{vmatrix}\\ -\begin{vmatrix}\sin\theta\sin\phi&r\sin\theta\cos\phi\\\cos\theta&0\end{vmatrix}&\begin{vmatrix}\sin\theta\cos\phi&-r\sin\theta\sin\phi\\\cos\theta&0\end{vmatrix}&-\begin{vmatrix}\sin\theta\cos\phi&-r\sin\theta\sin\phi\\\sin\theta\sin\phi&r\sin\theta\cos\phi\end{vmatrix}\\ \begin{vmatrix}\sin\theta\sin\phi&r\cos\theta\sin\phi\\\cos\theta&-r\sin\theta\end{vmatrix}&-\begin{vmatrix}\sin\theta\cos\phi&r\cos\theta\cos\phi\\\cos\theta&-r\sin\theta\end{vmatrix}&\begin{vmatrix}\sin\theta\cos\phi&r\cos\theta\cos\phi\\\sin\theta\sin\phi&r\cos\theta\sin\phi\end{vmatrix}\end{pmatrix}}{r^2\sin\theta}$$

$$=\begin{matrix}\mathbf{E}^r\\ \mathbf{E}^\theta\\ \mathbf{E}^\phi\end{matrix}\begin{pmatrix}\sin\theta\cos\phi&\sin\theta\sin\phi&\cos\theta\\ \frac{\cos\theta\cos\phi}{r}&\frac{\cos\theta\sin\phi}{r}&-\frac{\sin\theta}{r}\\ -\frac{\sin\phi}{r\sin\theta}&\frac{\cos\phi}{r\sin\theta}&0\end{pmatrix}\xrightarrow[r=\rho]{\theta=\pi/2}\begin{pmatrix}\cos\phi&\sin\phi&0\\ 0&0&-\frac{1}{\rho}\\ -\frac{\sin\phi}{\rho}&\frac{\cos\phi}{\rho}&0\end{pmatrix}$$

Covariant metric tensor $g_{\mu\nu}$ is matrix product $g=J^T\cdot J$ of Jacobian and its transpose. OCC g's are diagonal.

Covariant: $g_{rr}=1,\quad g_{\theta\theta}=r^2,\quad g_{\phi\phi}=r^2\sin^2\theta,$ Contravariant: $g^{rr}=1,\quad g^{\theta\theta}=1/r^2,\quad g^{\phi\phi}=1/r^2\sin^2\theta,$

Assignment 8 Solutions (contd.)

Ex.2 *"Plopped" Parabolic Coordinate solutions* Consider the GCC(Cartesian) definition: $q^1=(x)^2+y$, $q^2=(y)^2-x$

(a) Does an analytic Cartesian coordinate definition $x^j=x^j(m)$ exist? Not a very useful one.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the appropriate graph on the following page sketch the unitary vectors at the point $(x=1,\ y=1)$ (Arrow) and at the point $(x=1,\ y=0)$. Where, if anywhere, are they OCC?

(d) Find and indicate where, if anywhere, are the singularities of this GCC.

$$\begin{pmatrix}\frac{\partial\bar{q}^1}{\partial x}&\frac{\partial\bar{q}^1}{\partial y}\\ \frac{\partial\bar{q}^2}{\partial x}&\frac{\partial\bar{q}^2}{\partial y}\end{pmatrix}=\begin{pmatrix}\bar{\mathbf{E}}^1\\ \bar{\mathbf{E}}^2\end{pmatrix}=\begin{pmatrix}2x&1\\-1&2y\end{pmatrix}$$

Inverting:

$$\begin{pmatrix}\frac{\partial x}{\partial\bar{q}^1}&\frac{\partial x}{\partial\bar{q}^2}\\ \frac{\partial y}{\partial\bar{q}^1}&\frac{\partial y}{\partial\bar{q}^2}\end{pmatrix}=\begin{pmatrix}\bar{\mathbf{E}}_1&\bar{\mathbf{E}}_2\end{pmatrix}=\frac{\begin{pmatrix}2y&-1\\1&2x\end{pmatrix}}{1+4xy}$$

$\det J=1+4xy=0$ when: $xy=-\dfrac{1}{4}$

$$g^{11}=\bar{\mathbf{E}}^1\bullet\bar{\mathbf{E}}^1=\begin{pmatrix}2x&1\end{pmatrix}\bullet\begin{pmatrix}2x&1\end{pmatrix}=4x^2+1$$

$$g^{12}=\bar{\mathbf{E}}^1\bullet\bar{\mathbf{E}}^2=\begin{pmatrix}2x&1\end{pmatrix}\bullet\begin{pmatrix}-1&2y\end{pmatrix}=2(y-x)$$

$$g^{22}=\bar{\mathbf{E}}^2\bullet\bar{\mathbf{E}}^2=\begin{pmatrix}-1&2y\end{pmatrix}\bullet\begin{pmatrix}-1&2y\end{pmatrix}=1+4y^2$$

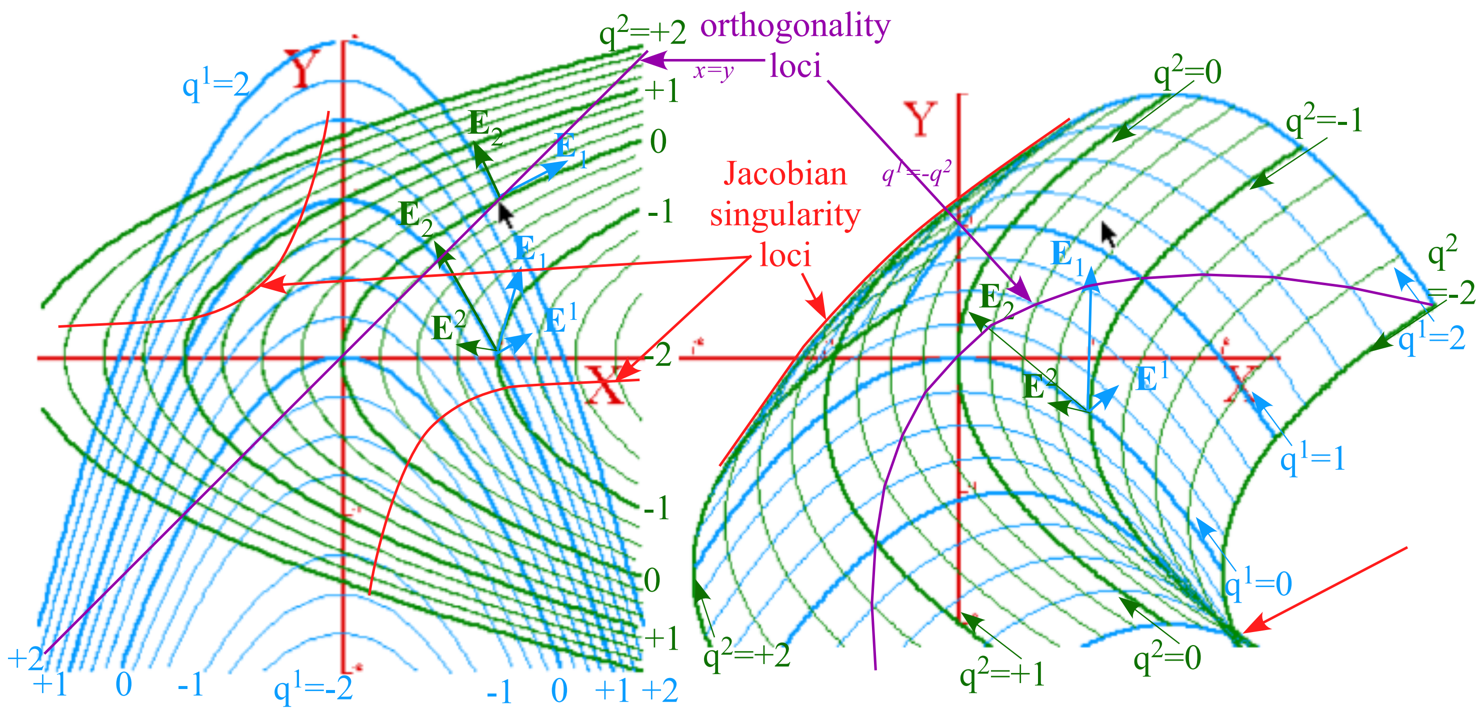
$$\det g=g_{11}g_{22}-g_{12}g_{12}=(\det J)^2=(1+4xy)^2$$

OCC where: $g^{12}=0=2(y-x)$ or: $y=x$

$$g_{11}=\bar{\mathbf{E}}_1\bullet\bar{\mathbf{E}}_1=\begin{pmatrix}2y\\1\end{pmatrix}\bullet\begin{pmatrix}2y\\1\end{pmatrix}\frac{1}{|g|}=\frac{4y^2+1}{(1+4xy)^2}$$

$$g_{12}=\bar{\mathbf{E}}_1\bullet\bar{\mathbf{E}}_2=\begin{pmatrix}2y\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2x\end{pmatrix}\frac{1}{|g|}=\frac{2(x-y)}{(1+4xy)^2}$$

$$g_{22}=\bar{\mathbf{E}}_2\bullet\bar{\mathbf{E}}_2=\begin{pmatrix}-1\\2x\end{pmatrix}\bullet\begin{pmatrix}-1\\2x\end{pmatrix}\frac{1}{|g|}=\frac{1+4x^2}{(1+4xy)^2}$$



"Plopped" Parabolic Coordinates *"Sliding" Parabolic Coordinates*

Assignment 8 (contd.) - solutions

Ex.3 *"Sliding" Parabolic Coordinates* Cartesian(GCC) definition: $x = 0.4\,(q^1)^2 - q^2,\quad y = q^1 - 0.4\,(q^2)^2$

(a) Does an analytic GCC coordinate definition $q^m = q^m(x^j)$ exist? $q^1 = const. \Rightarrow y = q^1 - 0.4(x - 0.4(q^1)^2)^2$

$$q^2 = const. \Rightarrow x = -q^2 - 0.4(y + 0.4(q^2)^2)^2$$

Not practical to solve quartic equation for q^1 or q^2.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

$$\begin{pmatrix} \frac{\partial x}{\partial q^1} & \frac{\partial x}{\partial q^2} \\ \frac{\partial y}{\partial q^1} & \frac{\partial y}{\partial q^2} \end{pmatrix} = \begin{pmatrix} \mathbf{E}_1 & \mathbf{E}_2 \end{pmatrix} = \begin{pmatrix} \frac{4}{5}q^1 & -1 \\ 1 & -\frac{4}{5}q^2 \end{pmatrix}$$

Inverting:

$$\begin{pmatrix} \frac{\partial q^1}{\partial x} & \frac{\partial q^1}{\partial y} \\ \frac{\partial q^2}{\partial x} & \frac{\partial q^2}{\partial y} \end{pmatrix} = \begin{pmatrix} \mathbf{E}^1 \\ \mathbf{E}^2 \end{pmatrix} = \frac{\begin{pmatrix} -\frac{4}{5}q^2 & +1 \\ -1 & +\frac{4}{5}q^1 \end{pmatrix}}{1 - \frac{16}{25}q^1 q^2}$$

$$\det J = 1 - \frac{16}{25}q^1 q^2 = 0 \text{ when: } q^1 q^2 = \frac{25}{16}$$

$$g_{11} = \mathbf{E}_1 \bullet \mathbf{E}_1 = \begin{pmatrix} \frac{4}{5}q^1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} \frac{4}{5}q^1 \\ 1 \end{pmatrix} = \frac{16}{25}(q^1)^2 + 1$$

$$g_{12} = \mathbf{E}_1 \bullet \mathbf{E}_2 = \begin{pmatrix} \frac{4}{5}q^1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -\frac{4}{5}q^2 \end{pmatrix} = \frac{-4}{5}(q^1 + q^2)$$

$$g_{22} = \mathbf{E}_2 \bullet \mathbf{E}_2 = \begin{pmatrix} -1 \\ -\frac{4}{5}q^2 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -\frac{4}{5}q^2 \end{pmatrix} = 1 + \frac{16}{25}(q^2)^2$$

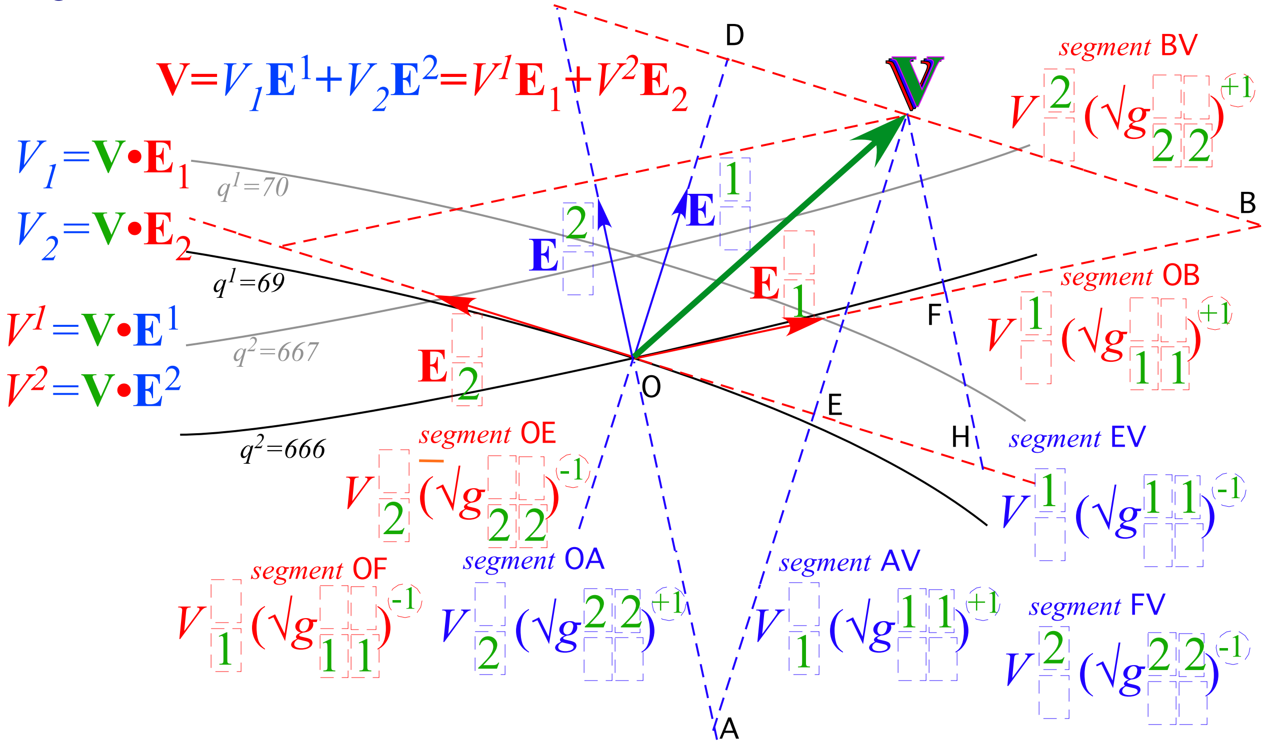
$$\det g = g_{11}g_{22} - g_{12}g_{12} = (\det J)^2 = \left(1 - \frac{16}{25}q^1 q^2\right)^2$$

$$g^{11} = \mathbf{E}^1 \bullet \mathbf{E}^1 = \frac{\begin{pmatrix} -\frac{4}{5}q^2 & 1 \end{pmatrix} \bullet \begin{pmatrix} -\frac{4}{5}q^2 & 1 \end{pmatrix}}{\det g} = \frac{+\frac{16}{25}(q^2)^2 + 1}{\left(1 - \frac{16}{25}q^1 q^2\right)^2}$$

$$g_{12} = \mathbf{E}^1 \bullet \mathbf{E}^2 = \frac{\begin{pmatrix} -\frac{4}{5}q^2 & 1 \end{pmatrix} \bullet \begin{pmatrix} -1 & +\frac{4}{5}q^1 \end{pmatrix}}{\det g} = \frac{\frac{4}{5}(q^1 + q^2)}{\left(1 - \frac{16}{25}q^1 q^2\right)^2}$$

$$g^{22} = \mathbf{E}^2 \bullet \mathbf{E}^2 = \frac{\begin{pmatrix} -1 & +\frac{4}{5}q^1 \end{pmatrix} \bullet \begin{pmatrix} -1 & +\frac{4}{5}q^1 \end{pmatrix}}{\det g} = \frac{1 + \frac{16}{25}(q^1)^2}{\left(1 - \frac{16}{25}q^1 q^2\right)^2}$$

Assignment 8 solution to Ex.**4** *GCC Coordinate diagram*.



Differential $d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial q^1}dq^1 + \frac{\partial \mathbf{r}}{\partial q^2}dq^2 = \mathbf{E}_1 dq^1 + \mathbf{E}_2 dq^2$ or approximation $\Delta\mathbf{r} \simeq \mathbf{E}_1\Delta q^1 + \mathbf{E}_2\Delta q^2$ shows how to scale covariant vectors $\mathbf{E}_1$ ($\Delta q^1 = 1, \Delta q^2 = 0$) and $\mathbf{E}_2$ ($\Delta q^1 = 0, \Delta q^2 = 1$). As sketched above, vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ approximately frame a "unit" parallelogram-grid-cell between points $(q^1 = 69, q^2 = 666)$, $(q^1 = 70, q^2 = 666)$, $(q^1 = 69, q^2 = 667)$, and $(q^1 = 70, q^2 = 667)$ separated by unit GCC difference $\Delta q^m = 1$. Of course the vectors would be better approximations of a smaller cell, say, a *nano unit cell* with $\Delta q^m = 10^{-9}$. Any consistent scale may be applied to draw $\mathbf{E}_m$-vectors since they have different units than the GCC q^m-coordinates themselves. But, then the contravariant $\mathbf{E}^m$-vectors must scale inversely so that $\mathbf{E}_m \bullet \mathbf{E}^m = 1$ and $\mathbf{E}_m \bullet \mathbf{E}^n = \delta^n_m$.

Assignment 8 solution to Extra credit Ex.**5** 3D-*GCC Coordinates*

4 The surface $z = f(x,y) = \frac{1}{2}x^2 + y^2$ is ($q^3=0$) part of a 3D GCC coordinate grid $q^1=x$, $q^2=y$, $q^3 = \frac{1}{2}x^2 + y^2 - z$ containing a projection of orthogonal *(x,y)* Cartesian coordinate grid. (That grid on the surface is obviously *not* orthogonal most places.)

a. Derive Jacobian *J(x,y)* and Kajobian *K(x,y)* for ($q^3=0$). b. Extract $\{\mathbf{E_1}, \mathbf{E_2}, \mathbf{E_3}\}$ and $\{\mathbf{E^1}, \mathbf{E^2}, \mathbf{E^3}\}$ in *(x,y,z)* basis.

Kajobian is easiest and derived first:

$$\begin{pmatrix} \frac{\partial q^1}{\partial x} & \frac{\partial q^2}{\partial x} & \frac{\partial q^3}{\partial x} \\ \frac{\partial q^1}{\partial y} & \frac{\partial q^2}{\partial y} & \frac{\partial q^3}{\partial y} \\ \frac{\partial q^1}{\partial z} & \frac{\partial q^2}{\partial z} & \frac{\partial q^3}{\partial z} \end{pmatrix} = \begin{array}{|ccc|} \mathbf{E}^1=\nabla q^1 & \mathbf{E}^2=\nabla q^2 & \mathbf{E}^3=\nabla q^3 \\ \hline 1 & 0 & x \\ 0 & 1 & 2y \\ 0 & 0 & -1 \end{array}$$

Inverse is Jacobian. It happens to be identical to Kajobian here!

$$\begin{pmatrix} \frac{\partial x}{\partial q^1} & \frac{\partial y}{\partial q^1} & \frac{\partial z}{\partial q^1} \\ \frac{\partial x}{\partial q^2} & \frac{\partial y}{\partial q^2} & \frac{\partial z}{\partial q^2} \\ \frac{\partial x}{\partial q^3} & \frac{\partial y}{\partial q^3} & \frac{\partial z}{\partial q^3} \end{pmatrix} = \begin{array}{|ccc|l} 1 & 0 & x & \mathbf{E}_1=\frac{\partial \mathbf{r}}{\partial q^1} \\ 0 & 1 & 2y & \mathbf{E}_2=\frac{\partial \mathbf{r}}{\partial q^2} \\ 0 & 0 & -1 & \mathbf{E}_3=\frac{\partial \mathbf{r}}{\partial q^3} \end{array}$$

c. Derive the *3-by-3* covariant metric $g_{\upsilon\nu}(x,y)$ and contravariant metric $g^{\upsilon\nu}(x,y)$ for ($q^3=0$) and tell which if any points on the surface have grids that are locally *orthogonal* and which if any are locally *orthonormal*.

The covariant and contravariant metrics are not identical. Only origin has orthogonality or orthonomality.

$$\begin{pmatrix} g^{11}=\mathbf{E}^1\bullet\mathbf{E}^1 =1 & g^{12}=\mathbf{E}^1\bullet\mathbf{E}^2=0 & g^{13}=\mathbf{E}^1\bullet\mathbf{E}^3=+x \\ 0 & g^{22}=\mathbf{E}^2\bullet\mathbf{E}^2=1 & g^{23}=\mathbf{E}^2\bullet\mathbf{E}^3=+2y \\ +x & +2y & g^{33}=\mathbf{E}^3\bullet\mathbf{E}^3 = 1+x^2+4y^2\end{pmatrix}\cdot \begin{pmatrix} g_{11}=\mathbf{E}_1\bullet\mathbf{E}_1=1+x^2 & g_{12}=\mathbf{E}_1\bullet\mathbf{E}_2=2xy & g_{13}=\mathbf{E}_1\bullet\mathbf{E}_3=-x \\ 2xy & g_{22}=\mathbf{E}_2\bullet\mathbf{E}_2=1+4y^2 & g_{23}=\mathbf{E}_2\bullet\mathbf{E}_3=-2y\\ -x & -2y & g_{33}=\mathbf{E}_3\bullet\mathbf{E}_3=1\end{pmatrix} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

d. Calculate and sketch covariant $\{\mathbf{E_1},\mathbf{E_2},\mathbf{E_3}\}$ on ($q^3=0$) surface at *(x=4,y=-2)* and *(x=3,y=+2)*.

e. Calculate and sketch contravariant $\{\mathbf{E^1},\mathbf{E^2},\mathbf{E^3}\}$ on ($q^3=0$) surface at *(x=4,y=+2)* and *(x=0,y=+4)*.*Assignment X-solutions*

"Unprofessional" Paraboloidal Coordinates (contd.)