Exercise Set 7 Due Tuesday Oct. 10: Based on Unit 1 Chapter 10 and 12 and Lectures 12-13 (2017).

"Professional" Parabolic and Hyperbolic Coordinates (Relates to Fig. 1.10.7)

1. Consider GCC definition: $q^1=\Phi= x^2 - y^2$, $q^2 =A= 2xy$. Both $(x^1=x,x^2=y)$ and $(q^1=u=\Phi,q^2=v=A)$ are Orthogonal Curvilinear Coordinates (OCC) related by an analytic function $w=z^2$ or $(u+iv)=(x+iy)^2$. You can treat *either one* as Cartesian. (This is based on the analytic function $f(z)=2z$ whose complex potential is ϕ=__________)

(a) Plot $(q^1=u,q^2=v)$ coordinate curves in a Cartesian $(x^1=x,x^2=y)$ graph. Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_k$ and $\mathbf{E}^k$ and metric tensors g_{mn} and g^{mn} for this GCC.
(b) Plot $(x^1=x,x^2=y)$ coordinate curves in a Cartesian $(q^1=u,q^2=v)$ graph. Derive the Jacobian, Kajobian, unitary vectors and metric tensors for this GCC.

Galaxy Grids

2. Consider the monopole field function $f(z)=e^{i\alpha}/z$ with complex source $e^{i\alpha}$ discussed in Lectures 13-14.

(a) Derive its $(q^1=\Phi,q^2=A)$ scalar and vector potential coordinate functions.
(b) Plot examples for angle $\alpha=30°$ and $\alpha=45°$.

Fun with Exponentials & more from The Story of e

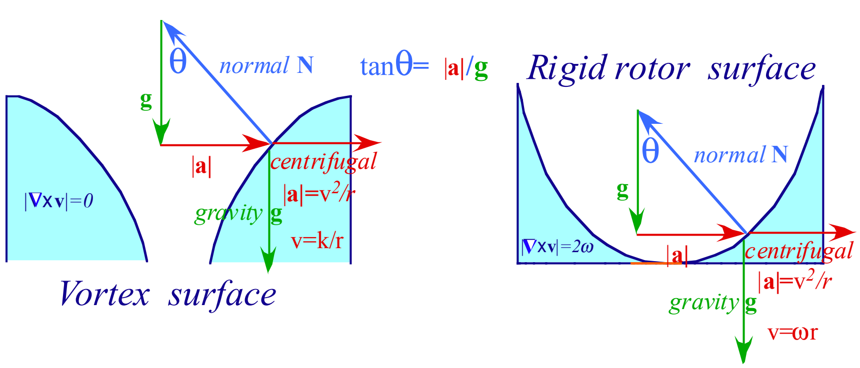
3. Consider a sequence of functions , $f_1(z)=z^z, f_2(z)=z^{f_1(z)}=z^{z^z}, f_3(z)=z^{f_2(z)}=z^{z^{z^z}}$,.... The function $f_N(z)$ has a finite limit $f_\infty(z)$ for N approaching infinity if argument z is small enough . ($z=1$ works! But, so does $z=\sqrt{2}$.)

(a) Find $f_\infty(\sqrt{2})=$____?
(b) Find an analytic expression for the limiting real z_{max} that involves the Euler constant. $e=2.718281828...$

Fun in the bathtub (This has a peculiar connection to "Sophomore-Physics-Earth" potential.)

4. Derive surface shape of rotating fluid subject to constraints on curl function $\nabla\times\mathbf{v}$ for velocity field. From this you should be able to derive surface altitude $S=S(r)$ as a function of radius r by relating balanced forces to differential slope. (Objects floating on these surfaces would not move up or down their $S(r)$ surface.)

(a) $\nabla\times\mathbf{v}=0$ (Whirlpool or Vortex) Complex vortex field $f(z^*)=v_x(x,y) +i\, v_y(x,y)=i/z^*$ has zero z-derivative and zero divergence (flux derivative $\nabla\bullet\mathbf{v} = 0$) and zero curl (circulation derivative $\nabla\times\mathbf{v}=\mathbf{0}$).
(b) $\nabla\times\mathbf{v}=const$. (Rigid rotation) Complex vortex field $f(z)=v_x(x,y) +i\, v_y(x,y)=i\omega\, z$ has constant imaginary z-derivative and therefore zero divergence (flux derivative $\nabla\bullet\mathbf{v} = 0$) and <u>constant</u> curl (circulation derivative $\nabla\times\mathbf{v}=\boldsymbol{\omega}$).



(c) How might the *"Sophomore-Physics-Earth"* potential be related to a surface whirlpool in deep water