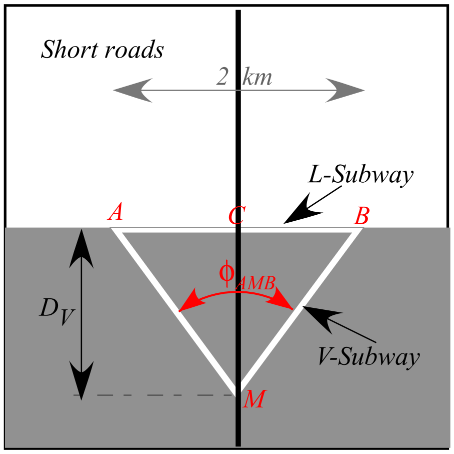
Assignment 6 - due Tue Oct. 03 - Mainly Chapters 9 , 11, and 12. Name____________

Geometric Optimization Exercises



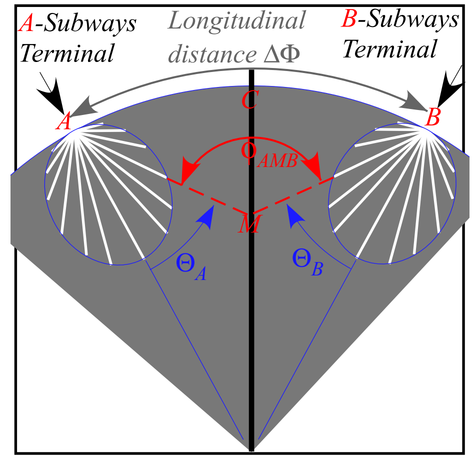


Fig. 1 Local friction-free subways on Sophomore Physics-Earth (SPE). Fig. 2 Global friction-free subways on SPE.

1. Assume *constant* gravity $g=9.8m/s^2$ for a friction-free local subway path AMB in Fig. 1 where turning vertex M is negotiated ideally with no loss of energy (or life). Derive depth D_V of point M and angle ϕ_{AMB} that gives the quickest trip from A to B ($AC=1km=CB$) thru M and derive the AMB time of travel τ_{AMB}.

2. Assume *Isotropic Harmonic Oscillator* IHO gravity in Fig. 2 with acceleration $g=9.8m/s^2$ at surface dropping to $g=0$ at Earth-center $C_\oplus$ (bottom-center of Fig.2). The objective (as in Ex. 1) is to find path AMB and angle ϕ_{AMB} having least time of travel τ_{AMB} between Terminal points A and B separated by great circle longitude angle $\Delta\Phi_{AB}$.
   Before solving main objective consider some alternative routes whose travel times should be easy to derive.
   a. Direct straight line route from A to B: $\tau_{A\ direct\ toB}$=______ *min*. Relate to SPE half-orbit period $\tau_\odot/2$
   b. Straight line segment routes A to C then C to B: $\tau_{AtoCtoB}$=______ *min*. " "
   c. Direct A to Earth-center $C_\oplus$ then $C_\oplus$ back up to B: $\tau_{AtoC_\oplus toB}$=______ *min*. " "

3. To solve main objective of Ex.2, imagine subway cars from terminal A or B leave their terminal at time $t_0=0$ and fall along straight tunnels (white lines) to positions at later time t_1 indicated by points on blue oval in Fig. 2. Ovals A and B expand equally until a touch-time t_T when they are tangent to each other and to vertical line CM.
   a. That touch-time t_T is related to total minimum travel time τ_{AMB}. How? (Recall τ_{AMB} for Ex. 1.)
   b. Derive polar $r(\Theta,t)$ equation for "ovals" relative to Terminal origin. What Thales geometric form is it?
   c. Relate optimizing angle ϕ_{AMB} to angle $\Delta\Phi_{AB}$ of longitudinal A to B separation. Plot geometry for $\Delta\Phi=\frac{\pi}{2}$.
   It helps to define a slope angle α between optimal subway path and terminal vertical radial line.
   d. A terminal-launched SPE circular orbit serves as a clock hand that quantifies growing "oval" size.
   Include this in your geometric plot to quantify optimal travel time τ_{AMB} and plot car positions vs. t.
   Show where cars are at fractions $\frac{1}{12},\frac{2}{12},\frac{3}{12},\ldots$ of half-orbit period $\tau_\odot/2$ to help find travel time τ_{AMB}.
   e. Verify geometry c and d in extreme cases of distance that is small ($\Delta\Phi\ll\frac{\pi}{2}$ like Ex.1) or large ($\Delta\Phi=\pi$).