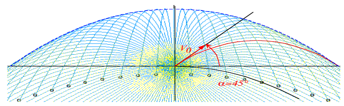
9/25/19 *Assignment 5 - due Wed Oct.2 - Chapters 9 - 12. "Families of orbits and contact envelopes."*



The atoms of NIST or volcanoes of Io

1. Suppose one of the volcanoes on Jupter's moon *Io* detonates in a constant gravity-$g(m\cdot s^{-2})$ vacuum sending equi-velocity $\pm v_0(m\cdot s^{-1})$ fragments off at initial elevation angles $\alpha=0°, 15°, 30°, 45°, 60°, 75°$, and $90°$ with the latter one going straight up to an altitude of $y=h_0=1$-*unit* on the attached plot 1 graph and then falling straight down.

(a.) That one distance unit has what *mks*-value in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1})$? h_0=_____________( ).

(b.) Derive the parabolic time-coordinates $x(t)$= _____________, $y(t)$= _____________ in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1})$ and elevation angle α.

(c.) Derive the parabolic focus-locus coordinates x_{foc}= _____________, y_{foc}= _____________ in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1}$ and elevation angle α *)* for $h_0=1$ and construct its curve on plot 1. This curve has Thales geometry (subtended angle of circle diameter or rectangle diagonal) that relate to trajectories. Show it on plots 1 to 4.)

(d.) Derive the parabolic directrix coordinate y_{dir}= _____________ in terms of $h_0=1$ and elevation angle α and construct this directrix line on graph for the cases $\alpha=0°$ to $90°$ listed above. Plot directrix of envelope, too.

(e.) Give general parabolic trajectory curve function $y(x)$= __________________ in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1})$ and α for $h_0=1$.

Four plots for four different launch angles 90°, 60°, 45°, and 30°

2. For cases $\alpha=0°, 30°, 45°, 60°$, and $90°$ construct curve points, tangents, kites, and contacts for $\alpha=60°$ on an attached $\alpha=60°$ plot 2, for $\alpha=45°$ on $\alpha=45°$ plot 3, and for $\alpha=30°$ on $\alpha=30°$ plot 4. *(Separate plots for clarity.)*

(a.) Locate the envelope contact points for the cases $\alpha=0°, 30°, 45°, 60°$, and $90°$ and construct enough of the envelope points and tangents to accurately represent the envelope on each of plots 2 to 4. If a contact point lies off a graph indicate where. Deduce $y_{envelope}(x)$= __________________ in terms of $h_0=1$.

(b.) Each parabola trajectory has kite-like structure (See Fig. 9.4.) as does the envelope. Draw and relate them.

(c.) Do any of the α-trajectories have the same shape as the envelope? If so, tell which one.

3. Now consider time behavior implicit in problem 1. In a "snapshot" of each moment, volcano fragments lie on "blast-front" curve. A geometric time unit T_1 is the time for the $\alpha=90°$ fragment to reach its peak.

(a.) That one time unit has what *mks*-value in terms of $g(m\cdot s^{-2})$ and $v_0(m\cdot s^{-1})$? T_1 =_______________( ).

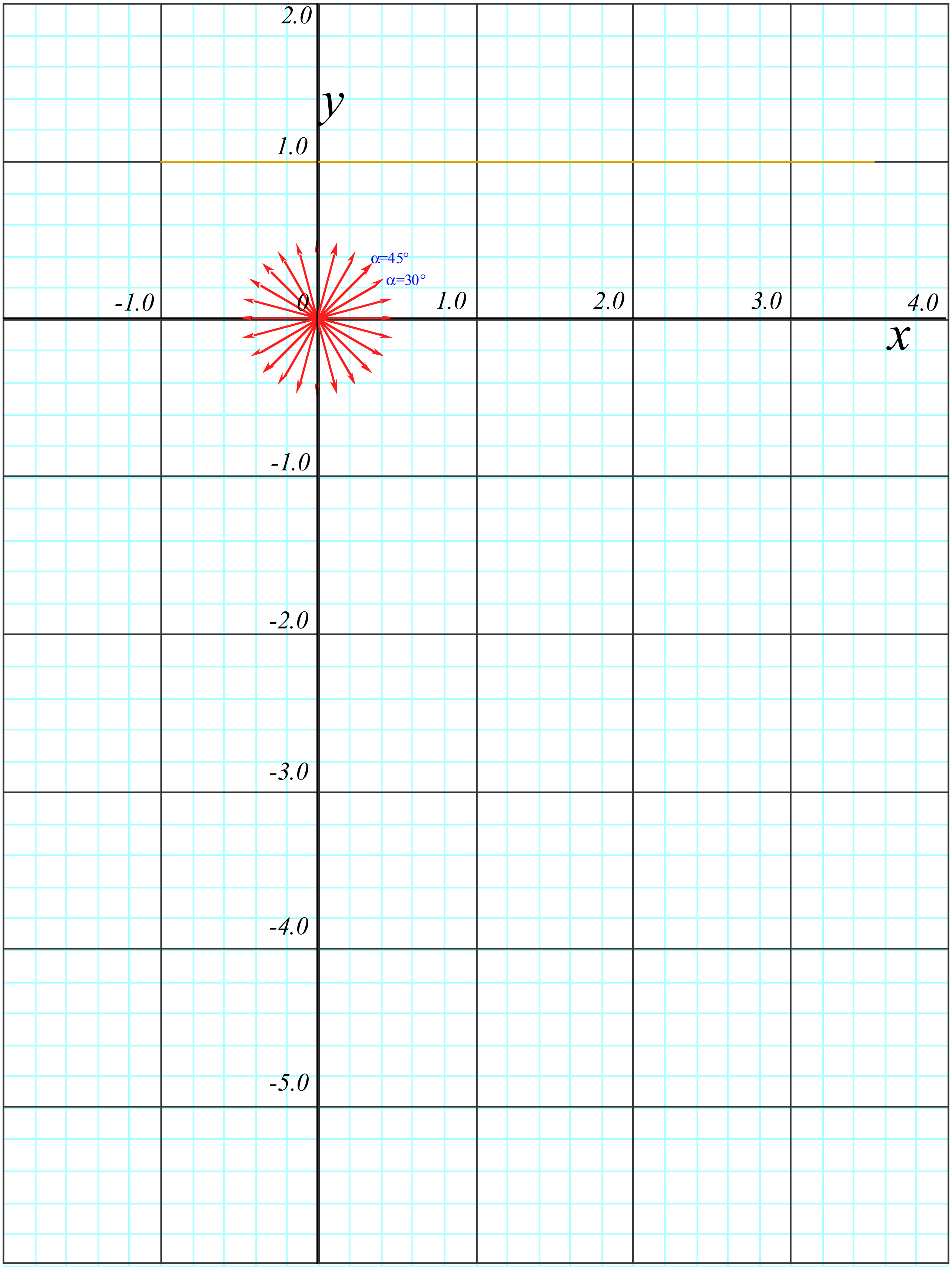
(b.) Give a brief explanation addressing why this "snapshot" curve or locus has to be (whichever): a parabola? ___ straight line?___ circle?____ ellipse?___(Check one and explain choice.)

(c.) Derive and/or construct the "blast-front" curve for the case $\alpha=90°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{90°}$=____ Find polar angle of contact normal.

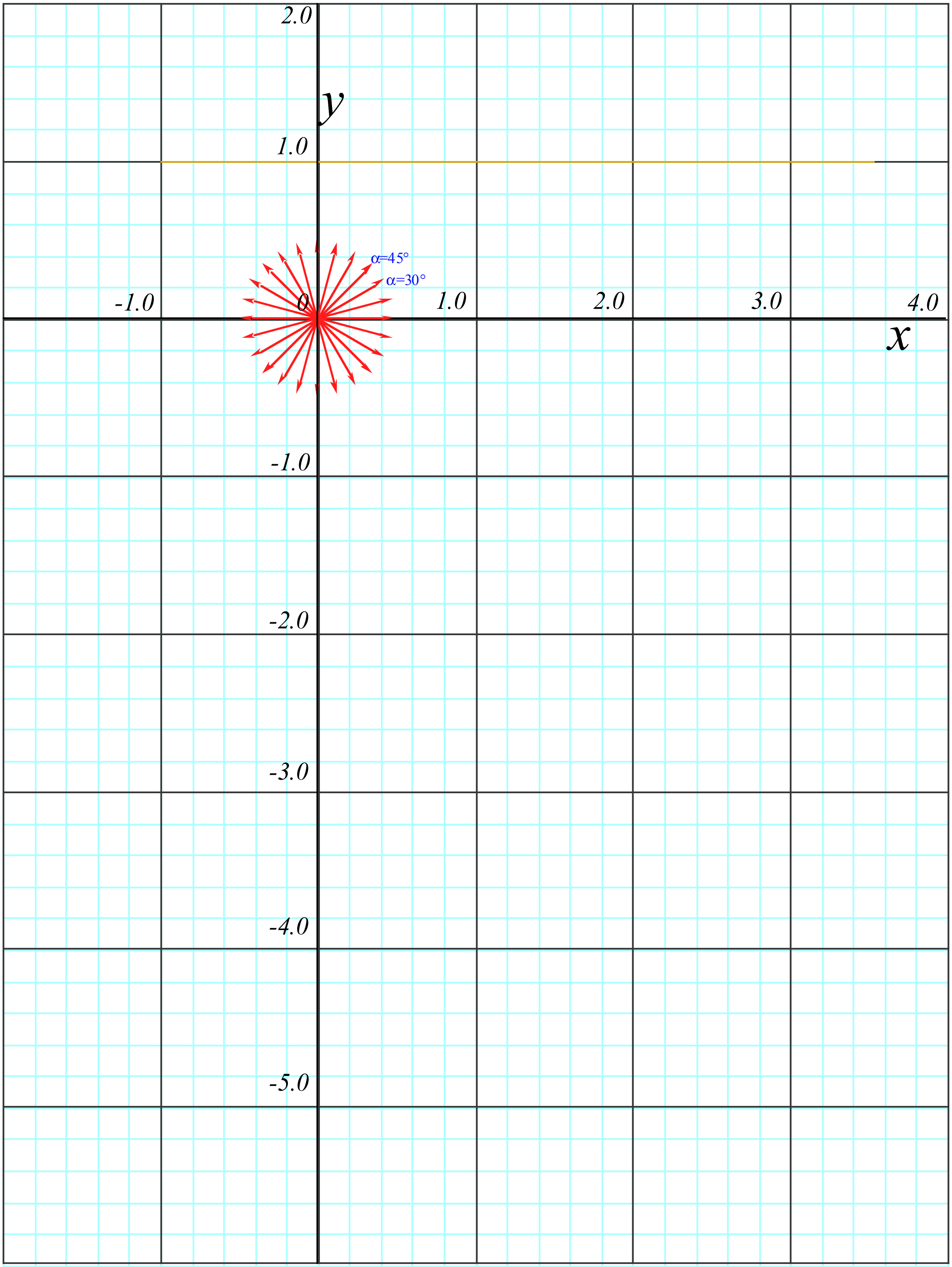
(d.) Derive and/or construct the "blast-front" curve for the case $\alpha=60°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{60°}$=____ Find polar angle of contact normal.

(e.) Derive and/or construct the "blast-front" curve for the case $\alpha=45°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{45°}$=____ Find polar angle of contact normal.

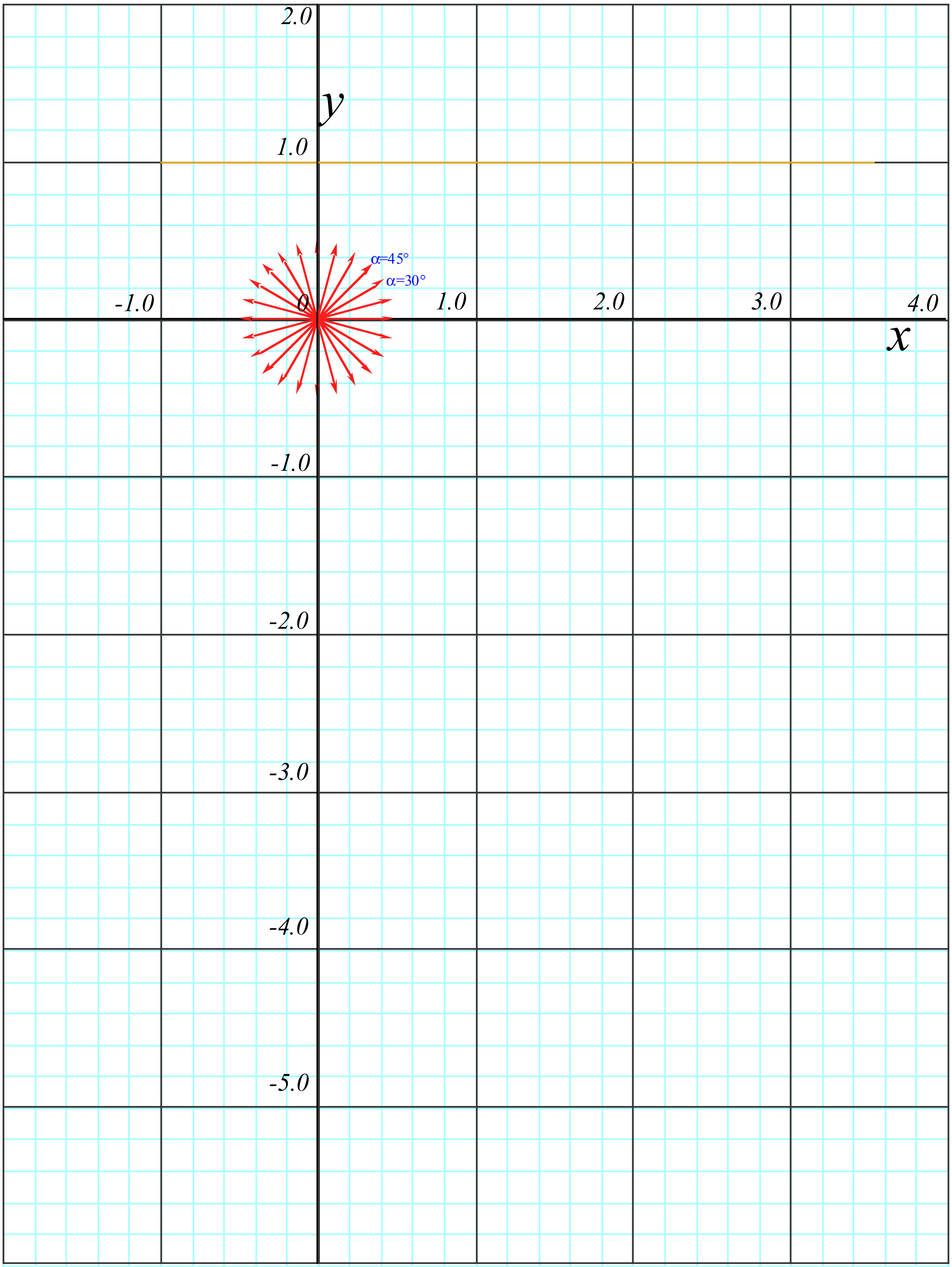
(f.) Derive and/or construct the "blast-front" curve for the case $\alpha=30°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{30°}$=____ Find polar angle of contact normal.



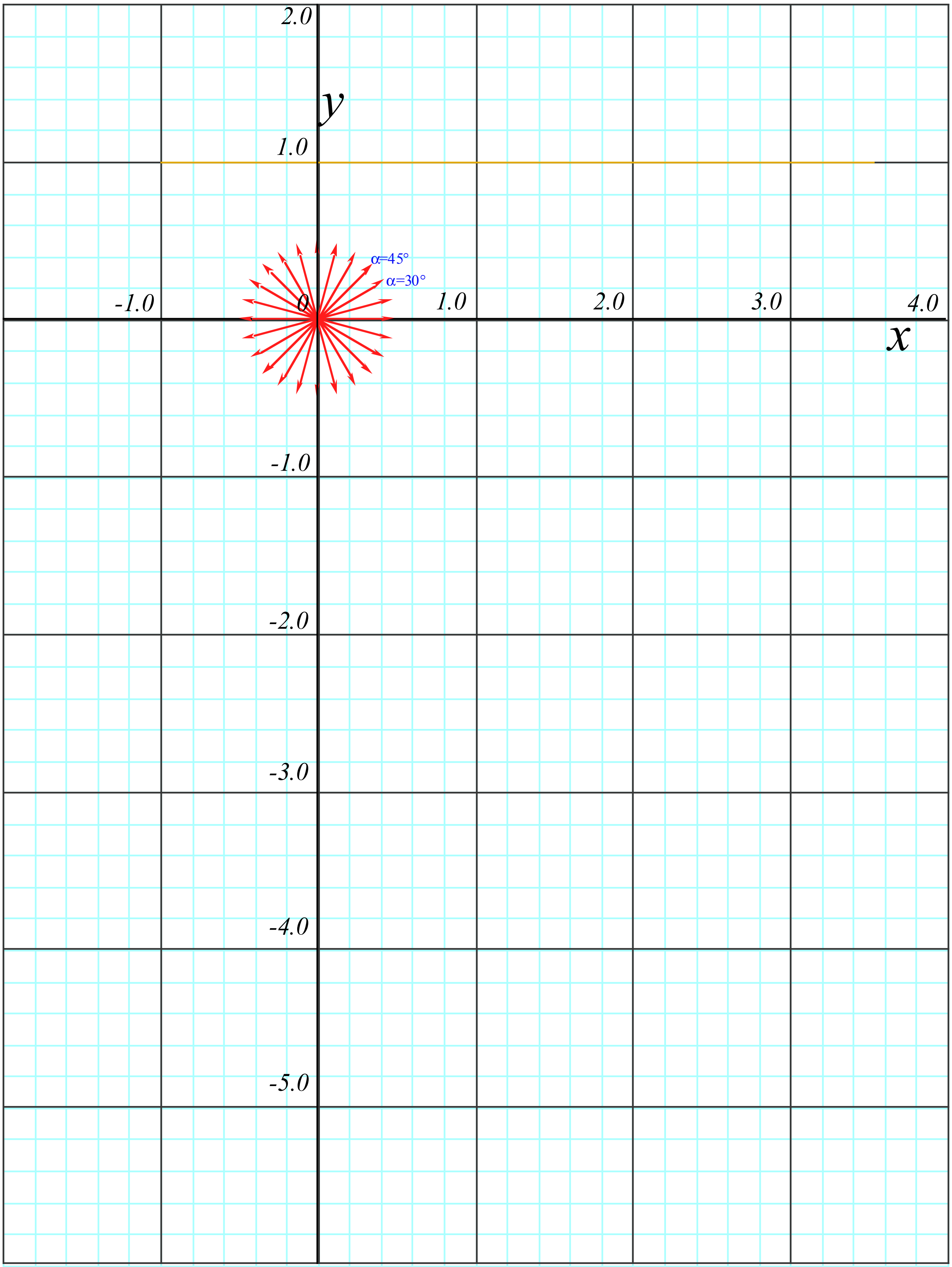
Plot 1: Geometry of α=90° path and α=0° path and where (if ever) they contact a "blast-front" .



2.Show geometry of α=60° path contacting envelope and "blast-front", kites, and foci of path and envelope. Show center of "blast-front" and its radius to contact point and its radius to intersection with α=0° path.



3.Show geometry of α=45° path contacting envelope and "blast-front", kites, and foci of path and envelope. Show center of "blast-front" and its radius to contact point and its radius to intersection with α=0° path.



4.Show geometry of α=30° path contacting envelope and "blast-front", kites, and foci of path and envelope. Show center of "blast-front" and its radius to contact point and its radius to intersection with α=0° path.