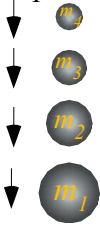


*Some lesser known properties of parabolic PE functions*

- 1.(a) Mechanics problems of atomic oscillators affected by electric fields is basic to spectroscopy. A useful model is potential  $V^{atom}(x)=kx^2/2$  function of center  $x$  of charge  $Q$  with polarizability spring constant  $k$ . A uniform electric field  $E$  applies force  $F=Q\cdot E$  to charge by adding potential  $V^E(x)$  to  $V^{atom}(x)$ . (Give  $V^E(x)=$ \_\_\_\_\_ and  $F^E(x)=$ \_\_\_\_\_) Consider the resulting potential  $V^{total}(x)$  for an atom for unit constants  $k=1$  and  $Q=1$ . Derive and plot the new values for equilibrium position  $x^{equil}(E)$ , energy  $V^{equil}(E)$ , dipole moment  $p^{equil}(E)=Q\cdot x^{equil}$ . Plot  $V^{total}(x)$  for field values of  $E=-3,-2,-1, 0, 1, 2,$  and  $3$ . Does frequency  $\omega^{equil}(E)$  vary with field  $E$ ? What curve do  $x^{equil}(E)$  points form?
- (b) Follow the steps to construct to external and internal potential energy  $V(r)$  and Force  $F(r)$  plots of the Sophomore-Physics Earth model. (Lect, 6 p.39-41 and p,62-65.) Describe the 3 equally spaced energy levels.

*Superball tower IBM model constructions (With initial  $V_k=-1$ )* See Fig. 8.1(b) p.103 of Text Unit 1 or Lect. 5 p.60



The 100% energy transfer limit (IBM values are  $v_1^{IN}=1$  and  $-1=v_2^{IN}=v_3^{IN}=v_4^{IN}=...$  after 1st floor bang.)

2. Suppose each  $m_k$  has just the right mass ratio  $m_k/m_{k+1}$  with the  $m_{k+1}$  above it to pass on all its energy to  $m_{k+1}$  so the top ball- $N$ , a 1gm pellet, goes off with the total energy. Construct velocity-velocity diagrams, indicate velocity at each stage, and derive the required intermediate mass values for (a)  $N=2$ , (b)  $N=3$ , (c)  $N=4$ .  
 (d) Give algebraic formula for this Maximum Amplified Velocity factor in terms of  $N$  ( $MAV(N)=$ \_\_\_\_\_?).  
 (e) Give algebraic formula neighbor-mass ratios  $R=M_{N-1}/M_N$  in terms of  $N$  ( $R(N)=$ \_\_\_\_\_?).

*N-Ball tower  $\infty$ -limits*

3. Suppose each  $m_k$  is very much larger than  $m_{k+1}$  above it so that final  $v_{k+1}$  approaches its upper limit. Then top  $m_N$  goes off with nearly the highest velocity  $v_N$  attainable. Construct the velocity-velocity diagrams. Indicate each intermediate velocity limit value at each stage and the limiting top value for (a)  $N=2$ , (b)  $N=3$ , (c)  $N=4$ .  
 (d) Give algebraic formula for Absolute Maximum Amplified Velocity factor in terms of  $N$  ( $AMAV(N)=$ \_\_\_\_\_?).

*The optimal idler (An algebra/calculus vs. geometry problem)*

- 4.(a) To get highest final  $v_3$  of mass  $m_3$  find optimum mass  $m_2$  in terms of masses  $m_1$  and  $m_3$  that will do that.  
 (b) Consider this problem in Galileo-shifted frame with:  $v_1^{IN}=2$  and  $0=v_2^{IN}=v_3^{IN}$  (Algebra simplifies for this.)  
 (c) Do V-V plots for case  $m_1=4$  and  $m_3=1$  (where  $m_2=$ \_\_\_\_?) ...for non-optimal case  $m_1=4, m_2=3, m_3=1$ .  
 (d) Give formula for optimal top mass final velocity in terms of  $m_1, m_2,$  and  $m_3$  and compare to result of 4(a). Plot that final velocity versus the idler mass  $x=m_2=0$  to 4. How sensitive is the optimal final  $v_3$  to  $x$ ?

*The backsides of exponentials*

5. *Some lesser known properties of exponentials and logarithms*

- (a) Do plots of exponential  $y=e^x$  and  $y=\log_e x$  functions on the same graph and draw any tangent-triangle whose hypotenuse is tangent to one of the curves and intercepts the  $x$  or  $y$  axis at integers  $-2, -1, 0, 1, 2,..$   
 (b) As a roller-coaster car moves down a track  $y=e^x$  it shines one laser beam along the track and another beam vertically down so both makes spots on baseline  $y=0$ . Find the distance between spots as function of  $x$ .