Name $\qquad$

## Pseudo-Rotations for Independent Bounce Model

Exercise 1 Estrangian plot in Fig. 5.2 (Details on p.30-33 of Lecture 3) has mass ratio $M_{1} / m_{2}=49 / 1$ and has nearly periodic path plot. (Experiment using BounceIt on web. http://www.uark.edu/ua/modphys/testing/markup/BounceltWeb.html) (Let the pen-mass be $m_{2}=1$ here.) Changing to $M_{1}=48.37$ gives more nearly periodic symmetry paths seen below.

(a) What order $N=$ $\qquad$ of $C_{N}$ or $D_{N}$ polygonal symmetry is appearing here?
(b) Give a closed formula for value of $M_{1}=48.37 \ldots$ (to 7 figures) that approaches exactly periodic behavior. Simplest formula should relate the tangent of a desired Estrangian rotation half-angle $\theta / 2$ to mass $M_{1}$.
(c) Ceiling height (It is $y_{\max }=7.0$ for cases above) may eventually affect or destroy periodicity.

Use BounceIt to show cases that are affected and discuss. (Many have chaotic behavior.)

## KE becomes PE

Exercise 2 A mass $m_{1}=1 \mathrm{~kg}$ ball is trapped (like Fig. 6.3) between two smaller mass $m_{2}=1 \mathrm{gm}$ balls of high speed $\left(v_{2}(0)=1000 \mathrm{~m} / \mathrm{s}\right.$ for $\left.x=0\right)$. Suppose this affects $m_{l}$ with an effective force law $F(x)$ of isothermal approximation (6.11). Assume $m_{l}$ motion is small and slow around $x=0$. ("Balls" idealize as point masses here.)
(a) A further approximation is the one-Dimensional Harmonic Oscillator (1D-HO) force and PE in (6.12). If each mass $m_{2}$ start in an interval $Y_{0}=1 m$, derive approximate 1D-HO frequency and period for mass $m_{1}$.
(b) What if the adiabatic approximation is used instead? Does the frequency decrease, increase, or just become anharmonic? Compare isothermal and adiabatic quantitative results for $m_{1}=1 \mathrm{~kg}$ ball being hit by two $m_{2}=1 \mathrm{gm}$ balls each having speed of $v_{2}(0)=1000 \mathrm{~m} / \mathrm{s}$ as each starts bouncing in a space of $Y_{0}=1 \mathrm{~m}$ on either side of the equilibrium point $x=0$ for the 1 kg ball.
(c) How does the frequency decrease or increase in isothermal case versus the adiabatic case if we shorten the run interval $Y_{0}=1 m$ to one-quarter meter?... What if we reduce the mass ratio $m_{l} / m_{2}$ by one-quarter?
(d) Derive the adiabatic frequency and period for the case $M=50 \mathrm{~kg}$ in adiabatic force of two $m=0.1 \mathrm{~kg}$ masses of initial speed $v_{0}=20 \mathrm{~m} / \mathrm{s}$ and range $Y_{0}=3 \mathrm{~m}$. Compare with Fig. 1.6.3c.

