Assignment 2 Read Unit 1 Chapters 1 thru 6. *Ex. 1.5.5 and 1.6.5 are due Thursday Sept. 1, 2016*

Reflections on reflections (A lesson in group representation theory)

Exercise 1.5.5 This exercise is intended to introduce matrix reflection and rotation operators and the groups they form. It involves the circular (V_1, V_2) plots (*"l'Etrangian space"*) introduced in Fig. 5.2b by eqs. (5.7)-(5.13) and the mirror diagrams in Fig. 5.3. All elastic (m_1 - m_2) collisions map an initial $\mathbf{V}^{IN}$=(V_1, V_2) vector into a $\mathbf{V}^{FIN}$ of the same length so all collisions map to unit vectors. All elastic m_1:m_2=3:1 collisions are described by a $D_6 \sim C_{6v}$ group of products of three reflection matrices **F**, **C**, and **M** described around *eq.* (5.3) of *Unit 1*. Note also *inversion operation* $\mathbf{F}\cdot\mathbf{C}=\mathbf{I}$ that commutes with all 12 operators in this $D_6 \sim C_{6v}$ group given in Lect. 3 p. 61 and described by operations as matrices and as reflections in (V_1, V_2) space.

(a) As far as the 2-D hex-plane is concerned the inversion **I** is a *rotation*. By how much?____° Explain.

(b) Note how each reflection like $\boldsymbol{\sigma}_{30}$ is labeled by giving angle 30° of a mirror-plane-slope and each rotation like $\mathbf{r}_{60}$ by the angle 60° it turns vectors. Show effect of $\boldsymbol{\sigma}_{30}$ and $\mathbf{r}_{60}$ on unit initial vectors (V_1, V_2)=(1, 0)=$\mathbf{e_x}$ and (0, 1)=$\mathbf{e_y}$. (This derives their matrix representations. See examples in Lect. 3 p. 56 thru p. 59.)

(c) Finish the $D_6 \sim C_{6v}$ group product table on page 61 of Lect. 3. First do sub-group $D_3 \sim C_{3v}$ whose table is upper-left *6-by-6* block of the $D_6 \sim C_{6v}$ table. Note that most C_{3v} elements do not commute with each other. (**ab ≠ ba**)

Then note each of the remaining *6-by-6* blocks follow a predictable pattern based on defining reflections $\boldsymbol{\sigma}$ as product $\boldsymbol{\sigma}=\mathbf{I}\cdot\mathbf{r}=\mathbf{r}\cdot\mathbf{I}$ of inversion **I** and C_{3v} rotation **r**. (This shows that C_{6v} reduces to outer product $C_{6v} = C_{3v} \times C_i$ of subgroup C_{3v} with inversion group $C_i=\{\mathbf{1}, \mathbf{I}\}$ since C_i commutes with C_{3v}.)

KE becomes PE

Exercise 1.6.5 In Fig. 6.3 (See also Lect. 4 p.45 to 48.†) a mass $m_1=50kg$ ball is trapped between two smaller mass $m_2=0.1kg$ balls of relatively high speed ($v_2(0)=20m/s$ at $t=0$) that provide m_1 with an effective force law $F(x)$ based on isothermal approximation (6.11). We assume m_1 moves only moderately far or fast from equilibrium at $x=0$. (We idealize "balls" as point masses here and in many other CM problems.)

(a) A further approximation is the one-Dimensional Harmonic Oscillator (1D-HO) force and PE in (6.12). If each mass m_2 starts in an interval $Y_0=3.5m$, derive isothermal approximate 1D-HO frequency and period for mass m_1.

(b) What if the adiabatic approximation is used instead? Does the frequency decrease, increase, or just become anharmonic? Compare isothermal and adiabatic quantitative results for $m_1=50kg$ ball being hit by two $m_2=0.1kg$ balls each having speed of $v_2(0)=20m/s$ as each starts bouncing in its space of about $Y_0=3.5m$ on either side of the equilibrium point $x=0$ for the $1kg$ ball. Which seems to give best approximation to Lect. 4 p.45-48† results?

† *Lect. 4 p.45-48 was upgraded after class with additional details.*

Unfinished group product table for *Exercise 1.5.5 from Lect. 3*

D_6	1	r_{120}	$\bar{r}_{120}$	σ_{60}	$\bar{\sigma}_{60}$	σ_z	I	$\bar{r}_{60}$	r_{60}	$\bar{\sigma}_{30}$	σ_{30}	$\bar{\sigma}_z$
1	1											
$\bar{r}_{120}$		1										
r_{120}			1									
σ_{60}				1								
$\bar{\sigma}_{60}$					1							
σ_z						1					$\bar{r}_{60}$	
I							1					
r_{60}								1				
$\bar{r}_{60}$									1			
$\bar{\sigma}_{30}$										1		
σ_{30}						r_{60}					1	
$\bar{\sigma}_z$												1

Note: $\bar{r}_{60} = Ir_{120} = r_{120}I = r_{-60}$ and: $I = r_{\pm 180}$

$\bar{r}_{120} = Ir_{60} = r_{60}I = r_{-120}$ and: $I^2 = 1$

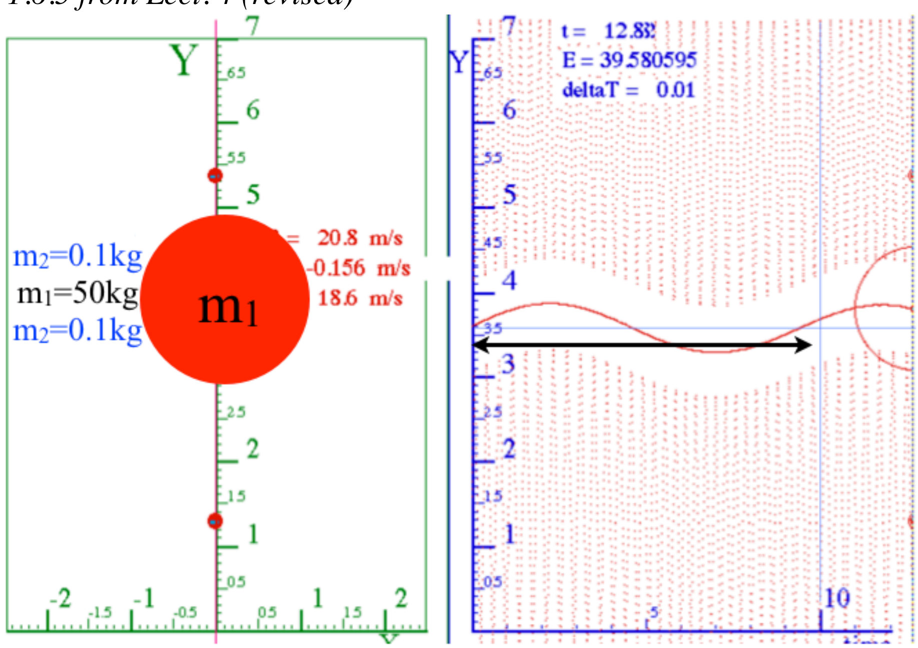
$\sigma_{60} = I\bar{\sigma}_{30} = \bar{\sigma}_{30}I$

$\bar{\sigma}_{60} = I\sigma_{30} = \sigma_{30}I$

$\bar{\sigma}_z = I\sigma_z = \sigma_z I$

For *Exercise 1.6.5 from Lect. 4 (revised)*

Switch m_1=m_3 with m_2 to match formula



Simulation of the adiabatic case

BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)

Sample problem: *Compute isothermal period given m_1= 50, m_2= 0.1= m_3, v_2= 20, Y_0= 3.5*

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Period :

$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

$=17.38$ *That's about √3 times too big!*

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$