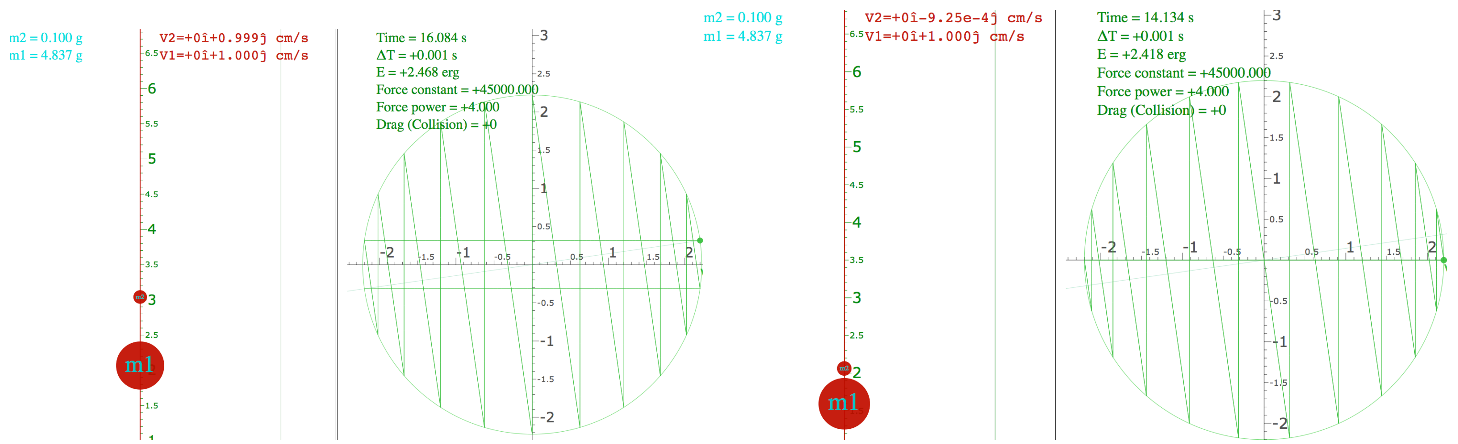
Due Tue. Sept. 7: Assignment 2- Read Unit 1 Chapters 4 thru 6 and do exercises

Exercise 1.5.3 Question about linear solution

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations. What is the *second* solution and to what simple process would it correspond? Describe algebraically and geometrically.

Pseudo-Rotations

Exercise 1.5.4 Estrangian plot in Fig. 5.2 (Details on p.30-33 of Lecture 3) has mass ratio $M_1/m_2 = 49/1$ and has nearly periodic path plot. (Experiment using BounceIt on web. http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html) (Let the pen-mass be m_2=1 here.) Changing to M_1= 48.37 gives better periodic paths shown below.



(a) Give a closed formula for value of M_1= 48.37...(to 7 figures) having *exactly* periodic behavior.

Simplest formula should relate the tangent of a desired Estrangian rotation half-angle $\theta/2$ to mass M_1 .

(b) Discuss the geometric interpretation of formula for periodic paths in general.

(c) Ceiling height (It is $y_{\max} = 7.0$ for cases above) may eventually affect or destroy periodicity.

Use BounceIt to show cases that are strongly affected and discuss. (Many have chaotic behavior.)

Reflections on reflections (A lesson in group theory)

Exercise 1.5.5 This exercise is intended to introduce matrix reflection and rotation operators and the groups they form. It involves the circular (V₁, V₂) plots (*"l'Etrangian space"*) introduced in Fig. 5.2b by eqs. (5.7)-(5.13) and the mirror diagrams in Fig. 5.3. All elastic (m_1 - m_2) collisions map an initial $\mathbf{V}^{IN}$=(V_1, V_2) vector into a $\mathbf{V}^{FIN}$ of the same length so all collisions map to unit vectors. All elastic m_1:m_2=3:1 collisions are described by a D_6~C_{6v} group of products of three reflection matrices **F**, **C**, and **M** described around *eq.* (5.3) of *Unit 1*. Note also *inversion operation* **F**·**C**=**I** that commutes with all 12 operators in this D_6~C_{6v} group given in Lect. 3 p. 54.

(a) You should verify the preceding statements with a few words and/or sketches.

(b) As far as the 2-D hex-plane is concerned the inversion **I** is a *rotation*. By how much?____° Explain.

(c) For mass ratio m_1:m_2=3:1 describe these operations as matrices and as reflections in (V_1, V_2) space. Note how reflections and rotations are labeled each by giving angle of a mirror-plane-slope or else of a rotation. Show their effect on unit initial vectors (V_1, V_2)=(1, 0)=$\mathbf{e_x}$ and (0, 1)=$\mathbf{e_y}$ and derive their matrix representations as done in Lect. 3 p. 49 thru p. 54.

(d) Form products of these to finish the D_6~C_{6v} group product table on page 54 of Lect. 3. First do sub-group D_3~C_{3v} whose table is upper-left *6-by-6* block of the D_6~C_{6v} table. Then note each of the remaining *6-by-6* blocks follow a predictable pattern based on defining reflections σ as product σ=**I**·**r** of inversion **I** and rotation **r**. (This shows that C_{6v} reduces to outer product $C_{6v} = C_{3v} \times C_i$ of subgroup C_{3v} with inversion group C_i={**1**, **I**}.)