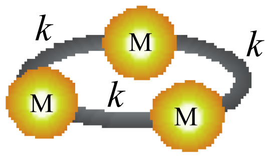
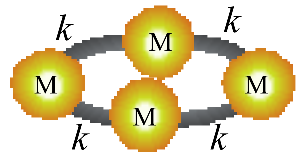
Assignment 12 - **Classical Mechanics 5103** 11/20/19 CMwBang Ch.4.4 to 4.8. and Lect.23-24 Due Tue. Nov. 26





Ex.1. *Two's Company but Three's a Crowd*

Three identical $M=1kg$ masses slide on a friction-free circular ring and are coupled by three identical $k=4N\cdot m^{-1}$ springs. (*Left hand sketch.*)

(a) Show that the resulting **K**-matrix can be written as a combination of three matrices that commute, satisfy $\mathbf{r}^{3m}=\mathbf{1}$, and therefore have 3rd-roots-of-unity eigenvalues $e^{im2\pi/3}=\{1, e^{i2\pi/3}, e^{-i2\pi/3}\}$.

$$\mathbf{r}^0=\mathbf{1}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\ \mathbf{r}^1=\mathbf{r}=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix},\ \mathbf{r}^2=\mathbf{r}\cdot\mathbf{r}=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$$

Note: $\mathbf{r}|1\rangle=|2\rangle$, $\mathbf{r}|2\rangle=|3\rangle$, $\mathbf{r}|3\rangle=|1\rangle$,*i.e.*, $\mathbf{r}\begin{pmatrix}1\\\cdot\\\cdot\end{pmatrix}=\begin{pmatrix}\cdot\\1\\\cdot\end{pmatrix}, \mathbf{r}^2\begin{pmatrix}1\\\cdot\\\cdot\end{pmatrix}=\mathbf{r}\begin{pmatrix}\cdot\\1\\\cdot\end{pmatrix}=\begin{pmatrix}\cdot\\\cdot\\1\end{pmatrix}$, etc.

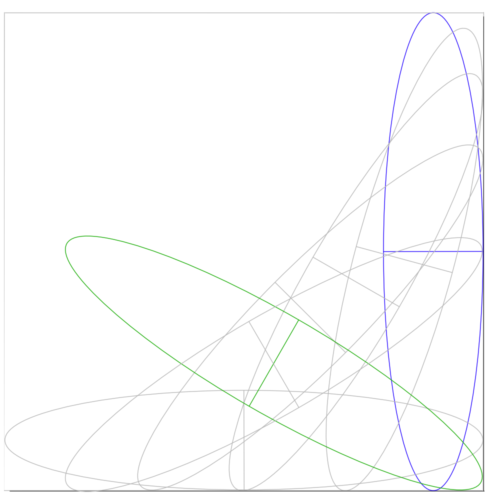
(b) Obtain a spectral decomposition of $\mathbf{r}^m$ and use it to get a **K**-matrix spectral decomposition, as well. Make a table of complex eigen-mode phasors and their frequencies. Show how both modes and frequencies relate to the 3rd-roots $e^{im2\pi/3}$ and plot the **K**-eigenvalues versus the mode number m. (This is called a dispersion plot, particularly if it's done for **H**-matrix eigenvalues.)

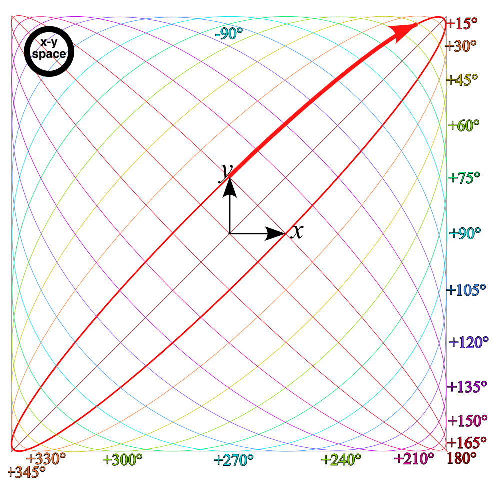
(c) Combining degenerate (equal-eigenvalue) complex eigenvector pairs to make pairs of eigenvectors with only real components.

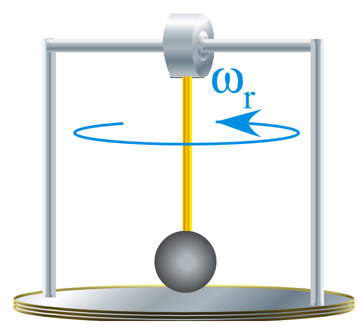
(d) Sketch the motions of each real eigenvector.

Ex.2. *...and Four's a Mob*

Do **Ex.1** for four identical spring-k-coupled masses. *(Right hand sketch above.)*







Ex.3 *Do you know your ellipse tangents?*

(a) Suppose an elliptic surfboard standing (like blue ellipse above) in a corner slides down the wall (like gray ellipses above). What curve does its center trace? Does it trace the same curve if it falls the other way (like green ellipse above) still tangent to wall and floor?

(b) This relates to energy of 2D IHO elliptic orbits plotted above-center. Does energy of orbit (Lect.7 p.58) depend on relative phase $\Delta\alpha$? If each ellipse shown above is boxed by its (2a-by-2b) rectangle what points or curve(s) do its corners trace as $\Delta\alpha$ varies? Construct these.

(c) Complete the labeling of the phase shift angles around top and left border. Discuss their physical and geometric significance.

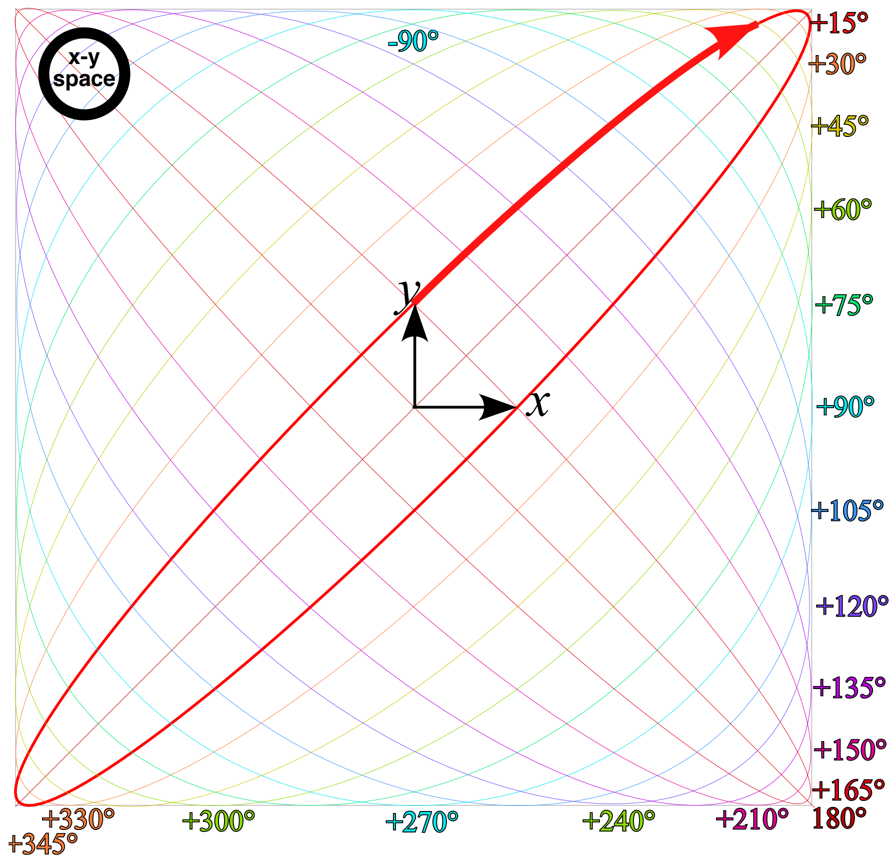
Pendulum on turntable (Soft-mode resonance)

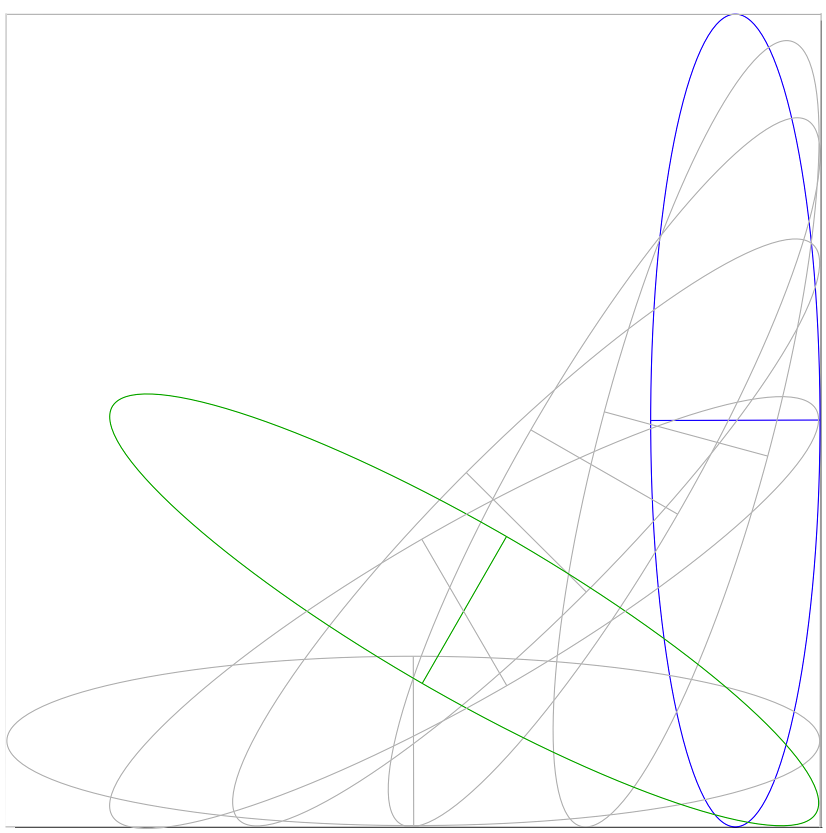
Ex.4 A pendulum supported by a circular ball bearing swings frictionlessly from a bearing secured to a turntable rotating at constant angular frequency ω_r with mass m at the end of a massless rod of length $\ell=1m$ Natural frequency of pendulum small-θ-angle motion at zero-ω_r in gravity (Say $g=10m/s^2$) is given by $\omega_\theta(\omega_r=0)=$____.

(a) Derive the Lagrangian and Hamiltonian using spherical coordinates in the rotating frame.

(b) Derive the θ-equilibrium points and small-oscillation frequency as a function of the frequency ω_r and ω_θ. Overlay plots of effective θ-potential for several key values of ω_r. What critical ω_r value makes $\theta=0$ angle unstable? Give pendulum frequency as function of g,ℓ, and ω_r before critical point and after the critical point.

(Use attached figures for plotting Ex.3 diagrams.)





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