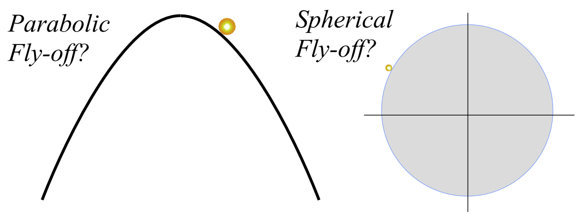
Assignment 11 Oct. 31,2018 Exercises due Wed. Nov. 7 Constrained motion theory in Unit 3 Ch.9 and Lect.19. Lecture 19 was not presented this year but last year's lecture is available online as is Unit 3 Ch. 9

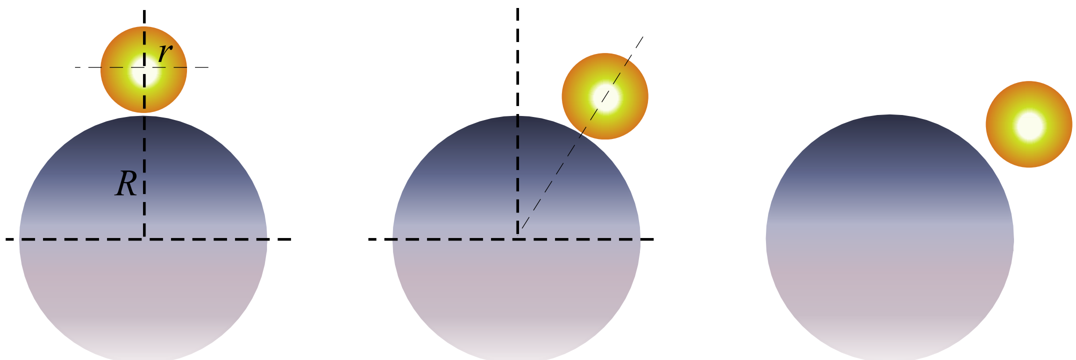


Parabolic Fly-off vs. Spherical Fly-Off

Ex.1. The frictionless constraint problem with mass m trapped in a parabolic well is shown to be an anharmonic oscillator in Sec. 3.9. Consider how m on a barrier might fall off under gravity $g=10m{\cdot}s^{-2}$.

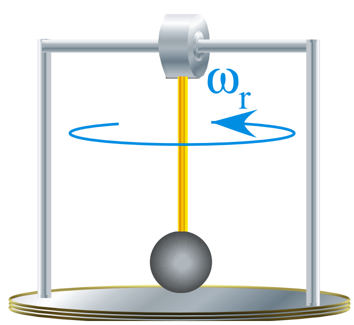
(a) Suppose an inverted parabolic road $y=-\frac{1}{2}kx^2$ with m starting with near-zero $v(0)$ at $x=0$ on top. Show whether there are x_{fly}, y_{fly}, and v_{fly} values where the mass m would fly off the road. Analyze and discuss.

(b) Do a similar analysis for a particle on a sphere of radius R. Compare to parabolic result of **(a)**.



"Easy as rolling off a log"

Ex.2. A ball of radius r and mass $m=1kg$ starting at the top of a fixed log of radius R and begins rolling down it. Assuming the sphere rolls without slipping calculate the angle from vertical where it last contacts the log. Give algebraic answers first. Then try $R=20cm$ and $r=1cm$ with $g=10m{\cdot}s^{-2}$, and then try $R=1cm$ and $r=20cm$. Compare these answers with each other and with those involving sliding particles in exercise **1(b)**.



Pendulum on turntable (Soft-mode resonance)

Ex.3 Suppose a pendulum supported by a circular ball bearing may swing without friction in the vertical plane of the bearing. The bearing plane is secured to a turntable that rotates at a constant angular frequency ω_r. The pendulum consists of a mass m at the end of a rod of length $\ell=1m$ and negligible mass with natural frequency of small θ-angle motion at zero-ω_r in gravity acceleration (Say $g=10m/s^2$) given by $\omega_0(\omega_r=0)=$____.

(a) Derive the Lagrangian and Hamiltonian using spherical coordinates in the rotating frame.

(b) Derive the θ-equilibrium points and small-oscillation frequency as a function of the frequency ω_r and ω_0. Overlay plots of effective θ-potential for several key values of ω_r. What ω_r value makes $\theta=0$ angle unstable?