Assignment 10 - **PHYS 5103**-11/06/19-Due Wed. Nov. 13 CMwBang! Ch 4.1 thru Ch.4.4. and Lectures 20-21

Ex.1 *The "standard" Lorentzian (Note: Review complex 2-pole potential $\phi(z)=1/z$ and $f(z)=-1/z^2$ (10.42) in Unit 1-Ch.10 Fig.10.11.)*

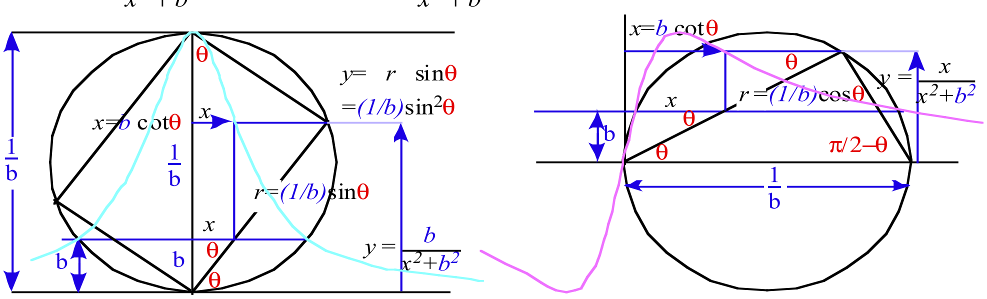
In physics literature, a standard Lorentzian function generally means a form $\text{Im}\,L(\Delta)=\Gamma/(\Delta^2+\Gamma^2)$ with constant Γ. In the *Near-Resonant Approximation* (NRA is (4.2.18) and (4.2.33)) the $L(\Delta)$ is a low Δ and Γ approximation to exact *G*-equations (4.2.15). A clear NRA derivation is given in Lect. 20 p. 49 to 53 and geometries of these NRA are sketched on p. 58 to 68.

(a) Reduce (4.2.15) to NRA $L(\Delta-i\Gamma)=\text{Re}\,L+i\,\text{Im}\,L=|L|e^{i\rho}$ functions of detuning "beat rate" $\Delta=\omega_s-\omega_0$, decay rate Γ, and phase lag angle ρ. Indicate what part of these expressions is the standard Lorentzian.

(b) Show that NRA for complex response *G*=Re *G* +iIm *G* gives circular arcs in the complex $\omega=|\omega|e^{i\theta}=|\omega|e^{i\rho}=\Delta+i\Gamma$ plane for constant decay rate Γ and variable detuning or beat rate Δ. How does this circle deviate from what is almost a circle in Fig. 4.2.6? (Consider higher Γ values for which NRA breaks down such as Fig. 4.2.14.) Relate to dipole scalar-Φ and vector-A potential field values plotted over coordinate lines for dipole force function $f(z)=1/z^2$ discussed in Ch. 10 of Unit 1. (See (10.42) and Fig. 10.11.)

(c) Do ruler-&-compass construction of NRA versions of the following Lorentz functions in figures below for $b=½$ and for $b=¼$. Construction is similar to that of IHO elliptical orbits (Unit 1 Fig. 3.6 p. 53 or Lect.7 p.22) in that it involves 90° points of a zig-zags.

$$\text{Re}\,G_{\omega_0}(\omega_s)=\frac{x}{x^2+b^2} \text{ and } \text{Im}\,G_{\omega_0}(\omega_s)=\frac{b}{x^2+b^2}$$.(See p. 58-68 of Lect. 20.)



(d) (*Xtra credit*)Study the Riemann-Cauchy equations for analytic function G* of Δ-iΓ that relate Δ and Γ partial derivatives of G^*_{Re} and G^*_{Im} (Recall Unit 1 eq.(10.32) or (better) Lect. 12 p.61) and consider what max our min values result from those derivatives being zero.

Ex.2 *Max and min G-values (Part (b-c) involves some derivative algebra!)*

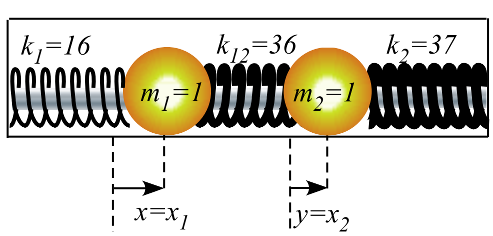
Derive equations for the extreme values for the *exact* Lorentz-Green response functions $G_{\omega_0}(\omega_s)$ as asked below.

Compare these to *Near-Resonant Approximations* (*NRA*) given in preceding **Ex.1**.Exact plots by calculator help to check algebraic answers.

(a₁) Find values which give maxima for: $\text{Re}\,G_{\omega_0}(\omega_s)$, $\text{Im}\,G_{\omega_0}(\omega_s)$, and $|G_{\omega_0}(\omega_s)|$ assuming ω_0 is constant and ω_s varies.

(a₂) Find values which give maxima for: $\text{Re}\,G_{\omega_0}(\omega_s)$, $\text{Im}\,G_{\omega_0}(\omega_s)$, and $|G_{\omega_0}(\omega_s)|$ assuming ω_s is constant and ω_0 varies.

Do (**a₁**) and (**a₂**) give the same results?



Ex.3 *Coupled oscillation by projection* **P***-operators*

Two identical mass $M=1kg$ blocks slide friction-free on a rod and are connected by springs $k_1=16N\cdot m^{-1}$ and $k_2=37N\cdot m^{-1}$ to ends of a box and coupled to each other by spring $k_{12}=36N\cdot m^{-1}$.

(a) Write Lagrangian equations of motion and derive a **K**-matrix form of them.

(b) Solve for eigenmodes and eigenfrequencies of system and plot their directions on an X,Y-graph. Use spectral decomposition methods (Lect. 21 p. 36-53 or Appendix 4.C) to derive eigensolution projectors and eigenvectors.

(c) Given initial conditions ($X(0)=1, Y(0)=0,$**V**$_0=0$), plot the resulting path in the XY-plane. Show it is a parabola.(*Tschebycheff* function)

(d) Use spectral decomposition (Lect. 21 or Appendix 4.C) to derive square-roots **H**=√**K**. (How many different square-roots does **K** have?)

(This is an important part of relating *Classical* coupled oscillators to *Quantum* coupled oscillators. See Lect. 22.)