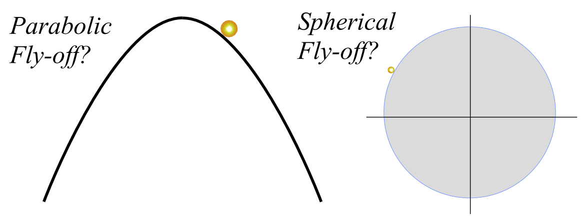
Assignment 10 Oct. 31,2017 Exercises due Tue. Nov. 7 Constrained motion theory in Unit 3 Ch.9 and Lect.19

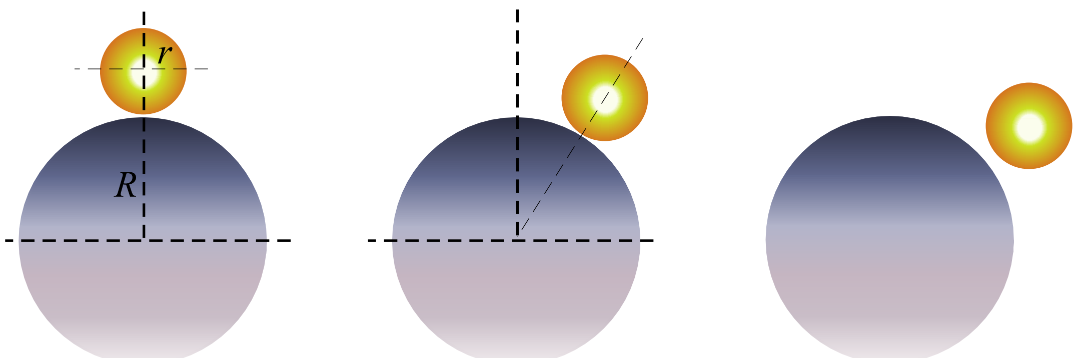


Parabolic Fly-off vs. Spherical Fly-Off

Ex.1. The frictionless constraint problem with mass m trapped in a parabolic well is shown to be an anharmonic oscillator in Sec. 3.9. Consider how m on a barrier might fall off under gravity $g=10m\cdot s^{-2}$.

(a) Suppose an inverted parabolic road $y=-\frac{1}{2}kx^2$ with m starting with near-zero $v(0)$ at $x=0$ on top. Show whether there are x_{fly}, y_{fly}, and v_{fly} values where the mass m would fly off the road. Analyze and discuss.

(b) Do a similar analysis for a particle on a sphere of radius R. Compare to parabolic result of **(a)**.



"Easy as rolling off a log"

Ex.2. A ball of radius r and mass $m=1kg$ starting at the top of a fixed log of radius R and begins rolling down it. Assuming the sphere rolls without slipping calculate the angle from vertical where it last contacts the log. Give algebraic answers first. Then try $R=20cm$ and $r=1cm$ with $g=10m\cdot s^{-2}$, and then try $R=1cm$ and $r=20cm$. Compare these answers with each other and with those involving sliding particles in exercise **1(b)**.

Xtra credit: For a given coefficient μ_S of stiction, find angle Θ_{slid} where rolling ball starts sliding.

Even more scary roller coasters

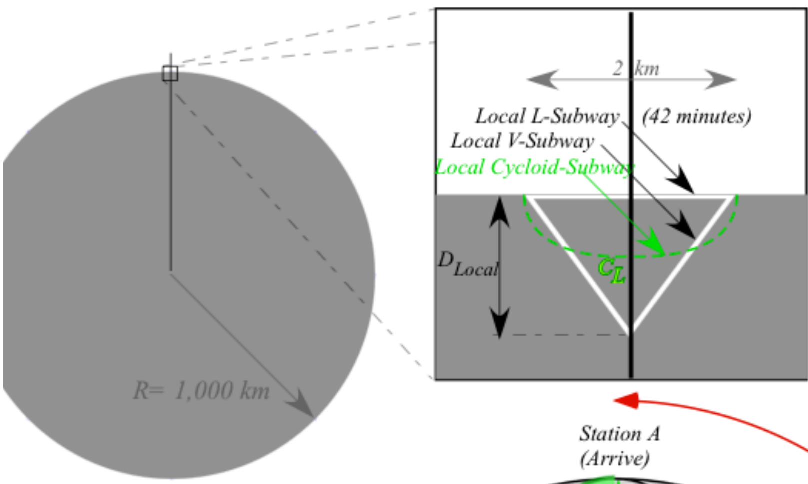
Ex.3. Sophomore Physics Earth (SPE) subways and scary cycloidal coasters are now rising from their graves (Asssignments 6 and 9) with ghostly problems involving hypo-cycloids and hyper-cycloids (See attached figures).

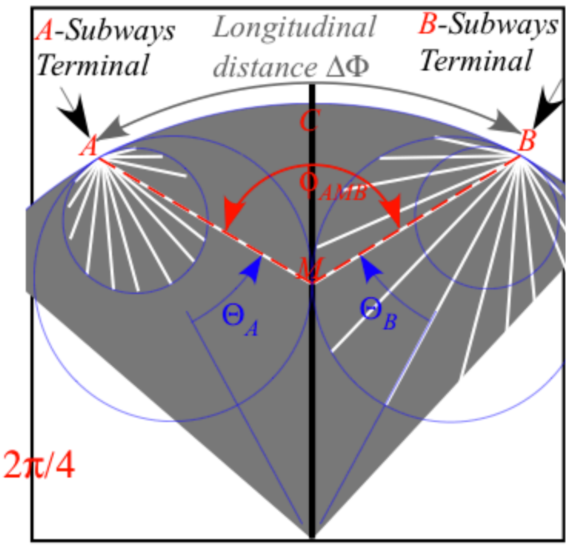
(a) First review the geometry of the optimal V-subway for a given longitudinal $\Delta\Phi$ separation angle and show to construct it simply by ruler and compass. Describe finding V and "kiss-arcs" for angle $\Delta\Phi$=30°, 60°, 90°, and 120°.

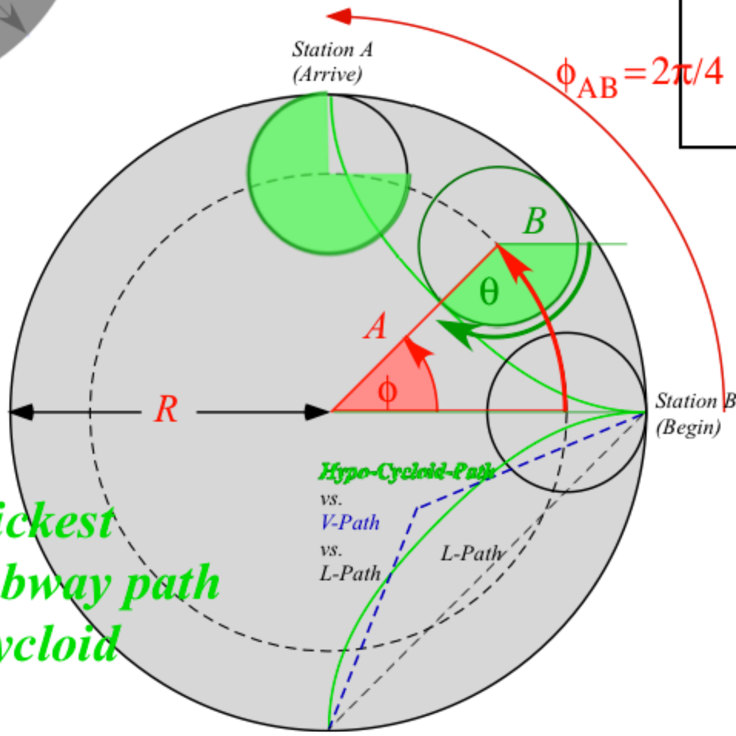
(b) Derive a formula for a hypo-cycloid made by a circle of radius r rolling inside a circle of radius R of an SPE. Attached figures may serve as a guide. Sketch resulting hypo-cycloid over optimal V for angles $\Delta\Phi$ in (b).

(c) Derive the equation motion for the subway that would follow an (r,R)-hypo-cycloid assuming initial and final velocity is zero at both initial and final points. Give simple formula for angular rates $\dot{\theta}$ and $\dot{\phi}$ in terms of fundamental angular frequency $\omega_\oplus$ of SPE. Compare 1-way travel times $\tau_\oplus/2=\pi/\omega_\oplus$ to the SPE time of *42min*.

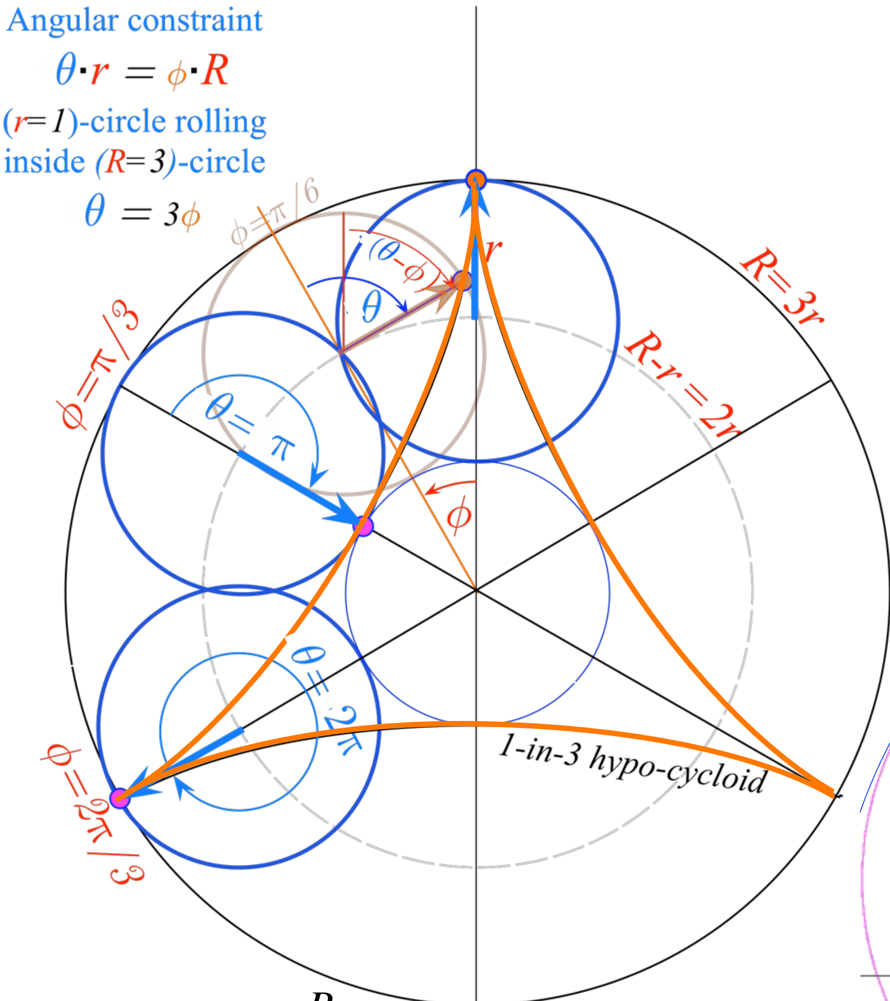
(d) Extra credit and possible AJP project. Discuss the dynamics of hyper-cycloidal constraint paths under a repulsive SPE IHO potential such as might result for a very rapidly rotating SPE.Would Coriolis accelerations alter the periods?

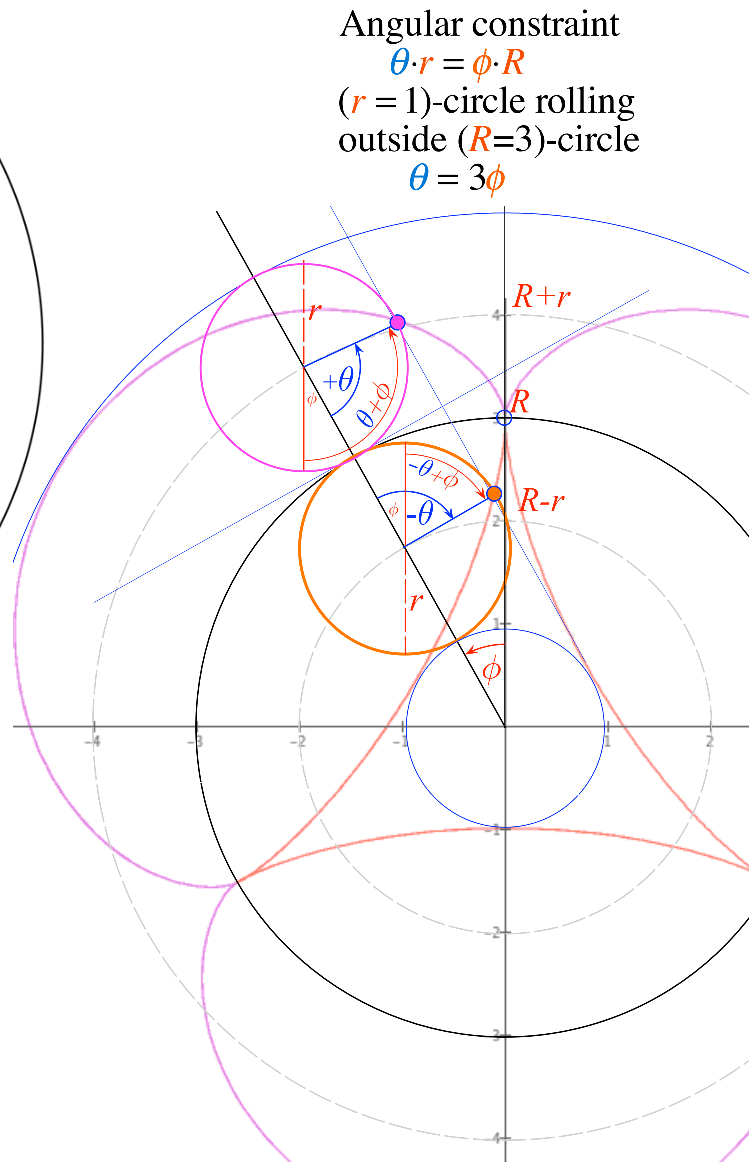






The very quickest planetary subway path is a Hypo-Cycloid





$$\theta r = R\phi \text{ or: } \theta = \frac{R}{r}\phi$$