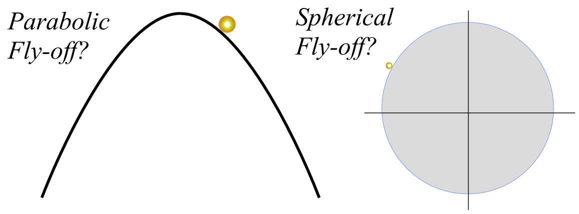
Assignment 10 Nov. 3,2016 problems Due Thur. Nov. 10

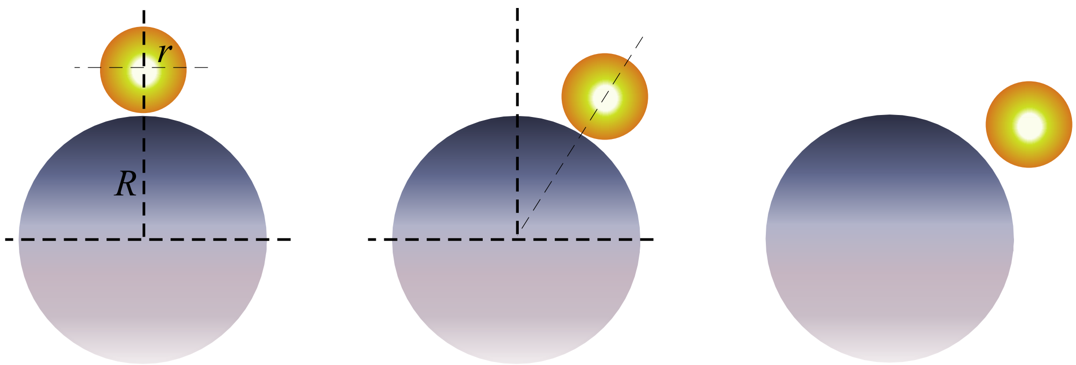


Parabolic Fly-off vs. Spherical Fly-Off

3.9.1 The frictionless constraint problem with mass m trapped in a parabolic well is shown to be an anharmonic oscillator in Sec. 3.9. Consider how m on a barrier might fall off under gravity $g=10m{\cdot}s^{-2}$.

(a) Suppose an inverted parabolic road $y=-\frac{1}{2}kx^2$ with m starting with near-zero $v(0)$ at $x=0$ on top. Show whether there are x_{fly}, y_{fly}, and v_{fly} values where the mass m would fly off the road. Analyze and discuss.

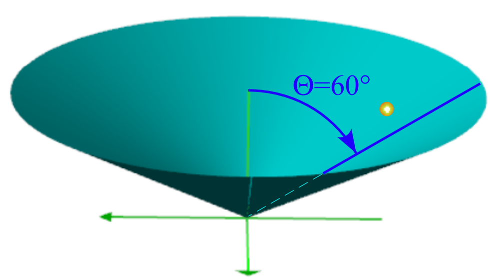
(b) Do a similar analysis for a particle on a sphere of radius R. Compare to parabolic result of (a).



"Easy as rolling off a log"

3.9.2 A ball of radius r and mass $m=1kg$ starting at the top of a fixed log of radius R and begins rolling down it. Assuming the sphere rolls without slipping calculate the angle from vertical where it last contacts the log. Give algebraic answers first. Then try $R=20cm$ and $r=1cm$ with $g=10m{\cdot}s^{-2}$, and then try $R=1cm$ and $r=20cm$. Compare these answers with each other and with those involving sliding particles in exercise 3.9.1(b).

Xtra credit: For a given coefficient μ_s of stiction, find angle Θ_{slid} where rolling ball starts sliding.



3.8.2. Funny orbits in a funnel

A mass is sliding frictionlessly in a circular $\Theta=60°$ cone with gravity $g\sim 10m{\cdot}s^{-2}$. *Caution: Use spherical coordinates.*

(a) Analyze orbits for the cone system as was done for the spherical fishbowl or "I-Ball" in constraint Lecture 19. For nearly circular orbits estimate radial and angular oscillation frequencies in terms of conserved quantities. Which closed orbit (if any) in Fig. 3.8.1 is closest to being achieved?

(b) Find Θ-cones that give closed orbits with frequency ratios given below and (if orbit is possible) sketch its path.

$\omega_r/\omega_\phi=1/1$: $\Theta_{1/1}=$___; $\omega_r/\omega_\phi=2/1$: $\Theta_{2/1}=$___; $\omega_r/\omega_\phi=3/1$: $\Theta_{3/1}=$___;

$\omega_r/\omega_\phi=1/2$: $\Theta_{1/2}=$___; $\omega_r/\omega_\phi=3/2$: $\Theta_{3/2}=$___; $\omega_r/\omega_\phi=1/3$: $\Theta_{1/3}=$___; $\omega_r/\omega_\phi=2/3$: $\Theta_{2/3}=$___.